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LOW-DIMENSIONAL SYSTEMS

Transport Properties of Two-Dimensional Hole Gas in a $Ge_{1-x}Si_x/Ge/Ge_{1-x}Si_x$ Quantum Well in the Vicinity of Metal–Insulator Transition[1]

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Abstract—Observation of a low-temperature transition from metallic ($\partial\rho/\partial T > 0$) to insulator ($\partial\rho/\partial T < 0$) behavior of resisitivity $\rho(T)$ induced by a perpendicular magnetic field B is reported for a two-dimensional (2D) hole system confined within Ge layers of a p-$Ge_{1-x}Si_x/Ge/Ge_{1-x}Si_x$ heterostructure. The essential feature of this system is that it is described by the Luttinger Hamilton with the g-factor highly anisotropic for orientations of magnetic field perpendicular and parallel to the 2D plane ($g_\perp \gg g_\parallel$). The positive magnetoresistance revealed scales as a function of B/T. We attribute this finding to suppression of the triplet channel of electron–electron (hole–hole) interaction due to Zeeman splitting in the hole spectrum.

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1. INTRODUCTION

For the diffusive motion of electrons in disordered conductors, the quantum corrections to the Drude conductivity σ_0 appear due to both single-particle weak localization (WL) effects and disorder modified electron–electron interaction (EEI) [1, 2]. Recently, observation of an apparent metal–insulator in high mobility semiconductor heterostructures (see the pioneer work [3] and references [4, 5] for an extensive review) has provoked a breakthrough in the theory of EEI effects for two-dimensional (2D) disordered systems [6, 7]. A general theory of the interaction induced quantum corrections to the conductivity tensor of 2D electrons is developed at $kT \ll \varepsilon_F$ for an arbitrary relation between kT and $\hbar/\tau$ (where τ is the elastic mean free time and ε_F is the Fermi energy), in the whole range of temperatures from diffusive ($kT\tau/\hbar \ll 1$) to the ballistic ($kT\tau/\hbar \gg 1$) regimes both for short-range (point-like) [6] and long-range (smooth) [7] random impurity potentials.

According to these latest theories, a linear increase of resistivity ρ with temperature in high-mobility Si-MOSFETs at large values of $\sigma \gg e^2/h$, which for a decade has been considered a signature of the "anomalous metallic" state, can now be described quantitatively in terms of interaction effects in the ballistic regime [8]. But the non-monotonous temperature dependence of $\rho(T)$ near the conjectural conductor–insulator transition (at $\sigma > e^2/h$) [8–10] does not yet have a generally accepted understanding. It is the subject of our investigation, realized on p-$Ge_{1-x}Si_x/Ge/Ge_{1-x}Si_x$ heterostructures.

We also investigate magnetoresistance in a perpendicular to the 2D plane magnetic field B where both the Zeeman splitting and weak localization dephasing effects should be taken into account. We extensively use some ideas exploited for interpretation of the experimental data on $\rho(B, T)$ dependencies of samples with parameters in a vicinity of conjectural conductor–insulator transition in high-mobility 2D semiconductor systems [9, 11–13].

2. EXPERIMENTAL RESULTS AND DISCUSSION

Transport properties of the multilayer p-$Ge/Ge_{1-x}Si_x$ heterostructures with the quantum well (Ge layer, QW) width $d_w \gtrsim 150$ Å and hole density per layer $p_s \gtrsim 3 \times 10^{11}$ cm^{-2} have been investigated in our previous works [14a, b]. Here the experimental data are presented and analyzed for two similar samples of the same system with narrower ($d_w = 80$ Å) Ge layers and lower $p_s = 1.1(1.4) \times 10^{11}$ cm^{-2}. Other structural parameters are: the number of ($Ge + Ge_{1-x}Si_x$) periods $N = 51$; $x \approx 0.20$; and the barrier ($Ge_{1-x}Si_x$) width $d_b = 150$ Å. This width

[1] The text was submitted by the authors in English.

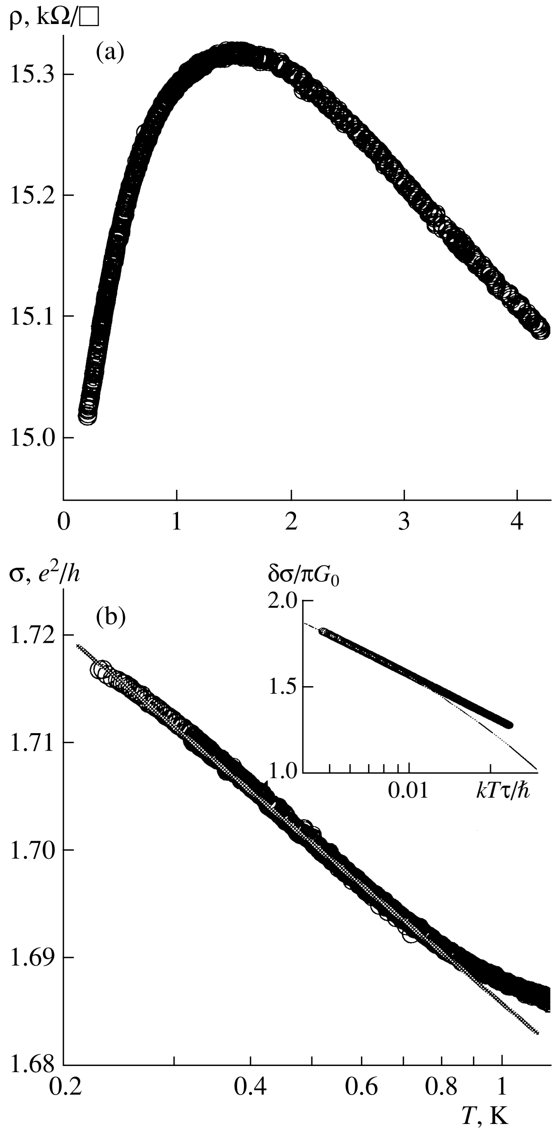

Fig. 1. (a) Temperature dependence of zero-field resistivity, (b) zero-field conductivity as a function of ln$T$. Solid line is the fit by Eq. (7) with $p = 1$ and $\lambda = 0.69$. Inset to Fig. 1b: points are experimental data for $T = 0.2$–1 K; the line is the result of calculations according to Eqs. (2), (3) with $\gamma_2 = 2.2$ ($F_0^\sigma = -0.69$).

is large enough to treat the barriers as impenetrable for holes and Ge QWs as independent, in accordance with our quantum Hall effect data. The hole mobility $\mu = 4.0(3.1) \times 10^3$ cm^2/(V s) [the latter and p_s obtained from zero-field resistivity $\rho_0 = 15(16)k\Omega/\square$ (per layer) and low field Hall $\rho_{xy}(B)$ at 4.2 K]. This gives $\varepsilon_F\tau/\hbar = 0.85(0.8)$. The central part of each barrier is doped with boron at concentration ~10^{17} cm^{-3} so that the undoped spacers of about 44 Å are on both its sides. The longitudinal and Hall resistivities have been investigated in magnetic fields configured perpendicular to the layers $B \le 5$T at temperatures $T = 0.2$–4.2 K. The data for the two samples are alike, so we shall concentrate on results for one on them. A brief description of the experimental results with preliminary interpretation has been done in the Materials of the 15th Ural International Winter School on the Physics of Semiconductors [14c].

In our previous works [14b] on samples of the same heterosystem with higher hole densities and mobilities ($k_F\lambda = 2\varepsilon_F\tau/\hbar \gtrsim 10$, k_F is the Fermi quasimoment, λ is the mean free path) the logarithmic drop of ρ has only been observed with temperature increases up to 20 K. For the samples investigated here, having $k_F\lambda \gtrsim 1$, a non-monotonous temperature behavior of zero-field resistivity is revealed (Fig. 1a). The "metallic" behavior ($\partial\rho/\partial T > 0$) takes place from 0.2 to 1.5 K and changes to the "insulating" behavior ($\partial\rho/\partial T < 0$) at higher temperatures. In the "metallic" region at $T \lesssim 1$ K, $\rho(T)$ dependence may be described by the logarithmic law (Fig. 1b).

Figure 2 shows magnetoresistance (MR) data at $T = 0.4$–0.9 K and $T = 4.2$ K. Note that *positive* magnetoresistance (PMR) is observed at all temperatures and the upturn of MR is sharper the lower the temperature (Fig. 2a). For the lowest temperatures $T \le 0.9$ K, the magnetoresistance curves scales as a function of B/T (Fig. 2b).

The condition $\varepsilon_F\tau/\hbar \approx 1$ for the samples investigated in this study formally indicates that we are near the critical region of the low-temperature transition from insulating to conducting behavior, which has been seen experimentally in a variety of high-mobility semiconductor systems [3–5]. We will take it into account in our analysis.

2.1. $\rho(B, T)$ Dependences at $T < 1$ K

It is essential that at $\varepsilon_F\tau/\hbar \approx 1$, the temperature range $kT \ll \varepsilon_F$ inevitably corresponds to the diffusive regime for EEI effect, $kT\tau/\hbar \ll 1$. Using the Shubnikov–de-Haas data for effective mass $m/m_0 = 0.08$ (m_0 is the free electron mass) [14a] we have Fermi energy $\varepsilon_F \approx 3.0$ meV and $\hbar/\tau \approx 3.5$ meV (the elastic mean free time $\tau \approx 1.9 \times 10^{-13}$ s) for the investigated sample. Then estimation gives $kT\tau/\hbar \approx 2.5 \times 10^{-2}T$ [K] and, hence, at $T \lesssim 1$ K we have: $kT\tau/\hbar \lesssim 0.025$ (for more detailed estimations see below).

The observed resistivity dependences $\rho(B, T)$ may be attributed to the quantum conductivity corrections due to both WL $\delta\sigma_{WL}$ and EEI $\delta\sigma_{ee}$. For the interaction effect at $B = 0$, we have [6]:

$$\delta\sigma_{ee} = \delta\sigma_c + \delta\sigma_T, \tag{1}$$

where

$$\delta\sigma_c = \frac{e^2}{2\pi^2\hbar}\left\{\frac{2\pi kT\tau}{\hbar}\left[1 - \frac{3}{8}f(T\tau)\right] + \ln\frac{kT\tau}{\hbar}\right\} \tag{2}$$

is the charge channel correction (which combines the Fock and singlet part of Hartree contributions) and

$$\delta\sigma_T = \frac{e^2}{2\pi^2\hbar} \times \left\{-3\gamma_2 \frac{2\pi kT\tau}{\hbar}\left[1 - \frac{3}{8}t(T\tau, \gamma_2)\right] - 3\lambda \ln\frac{kT\tau}{\hbar}\right\} \quad (3)$$

is the triplet channel correction (triplet part of the Hartree contribution). Contributions from the charge and triplet channels have different sign favoring localization or antilocalization, respectively.

Here the parameter γ_2 is the Fermi-liquid amplitude normalized by the density of states [9, 15],

$$\lambda = \frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2) - 1 \quad (4)$$

and the dimensionless functions $f(x)$ and $t(x, \gamma_2)$ describe the crossover between ballistic ($kT\tau/\hbar \gg 1$) and diffusive ($kT\tau/\hbar \ll 1$) regimes. In designations of [6]:

$$\gamma_2 = -\frac{F_0^\sigma}{1+F_0^\sigma}, \quad -1 < F_0^\sigma < 0.$$

In the diffusive approximation, electron–electron (e–e) quantum correction (the second terms in Eqs. (2), (3)) have been well documented earlier [1, 2, 15].

For WL effect at $B = 0$ and $\tau_\varphi \gg \tau$, we have [1, 2]:

$$\delta\sigma_{WL}(T) = -\frac{e^2}{2\pi^2\hbar}\beta\ln\frac{\tau_\varphi}{\tau}, \quad (5)$$

where $\tau_\varphi(T) \propto T^{-p}$ ($p > 0$) is a phase breaking time and prefactor β is 1 in the first approximation in $1/k_F\lambda$. In Eq. (5) we disregarded a mechanism of antilocalization due to the heavy and light hole states mixing developed in [16, 17] for p-type quantum wells with strong spin-orbit interaction, because of the small size of parameter $p_s d_w^2 = 0.07$ in our system. At intermediate and small values of $k_F\lambda$, the higher-order corrections to (5) should be taken into account. In a recent work [18] the terms of the second and third orders in $1/k_F\lambda$ are shown to cancel out in the interference correction at zero magnetic field. This means that the temperature dependence of $\delta\sigma_{WL}$ at $B = 0$ is just the same as for the case $k_F\lambda \gg 1$, down to low enough values of

$$\sigma = (2-3)\frac{e^2}{2\pi^2\hbar}$$

(see Eq. (5) with $\beta = 1$).

For the temperature dependent part of conductivity at $B = 0$, we then have from (2), (3) and (5):

$$\delta\sigma(T) = \delta\sigma_c(T) + \delta\sigma_T(T) + \delta\sigma_{WL}(T). \quad (6)$$

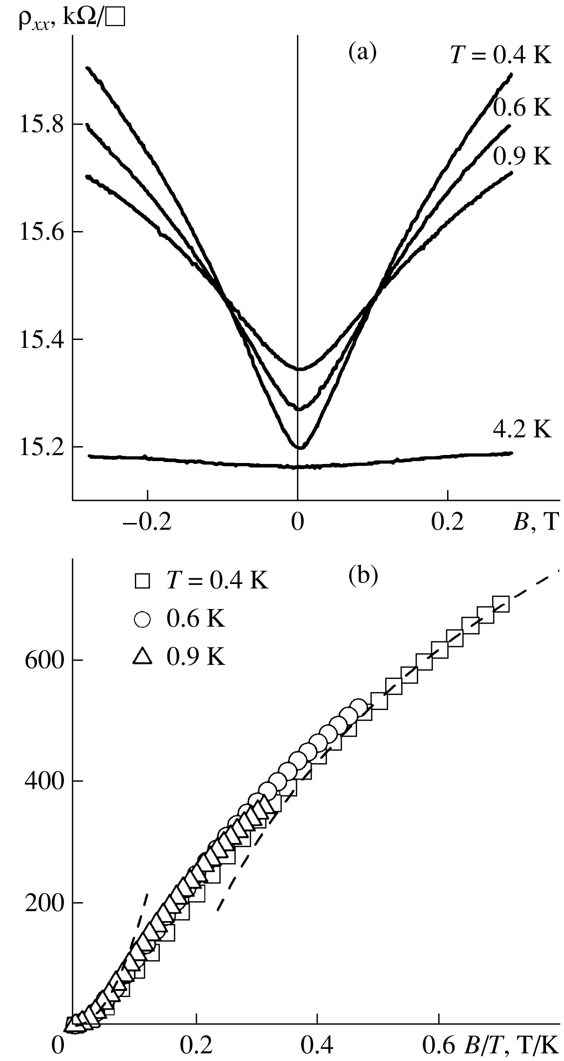

Fig. 2. (a) Dependences of resistivity on perpendicular to plane magnetic field at different temperatures, (b) magnetoresistivity as a function of B/T. Dashed lines are the fit by Eqs. (3) and (6) in the low field and high field limits.

Comparing the observed temperature dependence of σ in a region of "metallic" conductivity at $T < 1$ K (see the inset to Fig. 1b) with Eq. (6), we conclude that such behavior is evidence of the predominant role of the antilocalizing contribution by the triplet channel $\delta\sigma_T(T)$. From a fit of (6) with $p = 1$ [2, 19], $\beta = 1$ to the experimental curve, we have $\gamma_2 = 2.2$ or $F_0^\sigma = -\gamma_2/(1+\gamma_2) = -0.69$.

Now let us estimate the relative contributions of "ballistic" (linear in T) and "diffusive" (logarithmic) terms in our case. It follows from Eqs. (2) and (3) that at $T < 0.8$ K ($kT\tau/\hbar < 0.02$), the linear in T contributions amount to less than 4% for the charge channel and less than 8% for the triplet channel (with $F_0^\sigma = -0.69$). Hence, we are in effect at a diffusive limit in the low temperature region, $T \lesssim 1$ K.

By the way, in a diffusive approximation it is convenient to estimate the Fermi-liquid parameter γ_2 using the following simple expression [9]:

$$\frac{d\tilde{\sigma}}{d\xi} = \beta p + 1 - 3\lambda, \quad (7)$$

where $\tilde{\sigma} = (2\pi^2\hbar/e^2)\sigma$, $\xi = \ln(kT\tau/\hbar)$. Fitting Eq. (7) to the experimental points at $T < 1$ K (see Fig. 1b) gives $\lambda = 0.69$, and, according to Eq. (4), we have the same value of $\gamma_2 = 2.2$ with the corresponding accuracy ($\lesssim 10\%$).

The dependence of $\delta\sigma_{WL}$ on a perpendicular field at $B \ll B_{tr}$, $B_\varphi \ll B_{tr}$ (with the so called transport field $B_{tr} = \hbar c/4eD\tau$, $B_\varphi = \hbar c/4dD\tau_\varphi$, D—diffusion constant) is described by the expression [20]:

$$\delta\sigma_{WL}(B) = \alpha\frac{e^2}{2\pi^2\hbar}\left[\Psi\left(\frac{1}{2} + \frac{B_\varphi}{B}\right) - \ln\frac{B_\varphi}{B}\right], \quad (8)$$

with the Digamma function $\Psi(x)$ (see also the note to Eq. (5) about a spin-orbit contribution).

The calculation to the first order in $1/k_F\lambda$ gives the prefactor $\alpha = 1$ (without the spin-orbit interaction and with no magnetic impurities) [20]. The second-order terms in $1/k_F\lambda$, both in the WL contribution and in the correction induced by the mutual effect of WL and e–e interaction, have been analyzed in [18]. It was shown that for $B_T \ll B \ll B_{tr}$ (where $B_T = B_\varphi x$, $x \equiv kT\tau_\varphi/\hbar$), the shape of the magnetic field dependence of $\delta\sigma_{WL}$ remains the same as in Eq. (8), but the prefactor α is modified in the following way:

$$\alpha = 1 - 2\frac{G_0}{\sigma}, \quad (9)$$

with $G_0 \equiv e^2/2\pi^2\hbar$. As it will be shown below, in our case we have $kT\tau_\varphi/\hbar \approx 1$ and $B_T \approx B_\varphi$. At $B \ll B_\varphi$ the prefactor $\alpha \approx 1$, as for $B = 0$ [18].

For our sample $B_{tr} \approx 1.5$ T, so that at $B \le 0.3$ T the inequality $B \ll B_{tr}$ is rather well satisfied. Equation (8) leads to a *negative* magnetoresistance (NMR) due to suppression of WL by a magnetic field (dephasing effect). Note that $\delta\sigma_{WL}$ depends on the ratio B/B_φ and thus for $p = 1$ it is a function of only B/T.

In our systems with a large g-factor, spin-splitting effects should be taken into account. For the B-dependent (Zeeman) part of the total EEI contribution we have [1, 21–23]:

$$\delta\sigma_{ee}(B, T) \equiv \delta\sigma_z(B, T) = \frac{e^2}{2\pi^2\hbar}G(B/B_z), \quad (10)$$

with $B_z \equiv kT/g\mu_B$, where g is the electron Lande factor and μ_B is the Bohr magneton.

The function $G(B/B_z)$ in Eq. (10) describes the effect of Zeeman splitting of EEI that leads to a PMR due to suppression of a great part of the antilocalizing triplet contribution into $\delta\sigma_{ee}$. The expression for it in the diffusive approximation was first deduced by Lee and Ramakrishnan for weak EEI ($\gamma_2 \ll 1$) [21] and then by Castellani, Di Castro and Lee for any value of γ_2 [22]. At present, the $G(B/B_z)$ expression for arbitrary strength of interaction is derived anew, both in diffusive and ballistic regimes [23].

It is important that, according to [23] (see formulas (9), (10) of that reference), the argument of the G-function, $B/B_z = g\mu_B B/kT$, depends only on the bare electron g-factor, which is not renormalized by the Fermi liquid EEI (for a given semiconductor, it is the effective electron g-factor). Hole gas in Ge quantum wells for investigated p-Ge/Ge$_{1-x}$Si$_x$ heterostructures is described by a Luttinger Hamiltonian with effective g-factor highly anisotropic relative to mutual orientation of magnetic field and 2D plane: at the bottom of the ground hole subband $g_\perp = 6\kappa \approx 20.4$ (where the Luttinger parameter $\kappa \approx 3.4$ for Ge [24]) and $g_\parallel = 0$ for magnetic fields perpendicular ($B_\perp$) and parallel ($B_\parallel$) to 2D plane, respectively [25, 26].

We have found that a dependence of ρ on magnetic field, namely, on a ratio B/T (see Fig. 2b), may be quantitatively described only by a combination of both PMR due to Zeeman splitting and NMR due to the WL dephasing effect, with some predominance of the first one. At $\omega_c\tau \ll 1$ for magnetoresistance $\Delta\rho_{xx} = \rho_{xx}(B, T) - \rho_{xx}(0, T)$, we have:

$$\Delta\rho_{xx}(B, T)/\rho_0 = -\delta\sigma(B, T)/\sigma_0$$

and fitting of formulas (8) and (10) to $\Delta\rho_{xx}(B/T)$ dependence gives an opportunity to estimate both the g-factor and the dephasing time τ_φ (at $B < 0.3$ T, $\omega_c\tau < 0.1$).

In a weak field limit $B \ll B_z$ [22, 23], $B \ll B_\varphi$ [20], we obtain for $\delta\sigma(B) = \delta\sigma_z(B) + \delta\sigma_{WL}(B)$ from (8) and (10):

$$\delta\sigma(B, T) = \frac{e^2}{2\pi^2\hbar}\left[-0.091\gamma_2(1+\gamma_2) + 0.042\left(\frac{B_z}{B_\varphi}\right)^2\right] \times \left(\frac{g\mu_B}{k}\right)^2\left(\frac{B}{T}\right)^2, \quad (11)$$

where the ratio B_z/B_φ is T-independent (see Appendix).

A fitting of (11) to experimental data with $\gamma_2 = 2.2$ and varying values of g-factor is presented in Fig. 3a.

It gives $B_z/B_\varphi = 3.7$, 3.8 and 3.9 for $g = 10$, 14 and 20, respectively. As seen, the parameter B_z/B_φ only weakly varies with variation of g.

From the position of a bending point on the $\delta\sigma(B, T)$ dependence (a position of the minimum for a first derivative $\partial\sigma(z)/\partial z$ with $z \equiv B/T$ on the inset of Fig. 3b), which is very sensitive to the g-factor value, we have an opportunity to completely define the parameter values. The best fit with experimental data gives $g = 14$ with accuracy of about ten percent. That, bearing in mind the weak field analysis, fixes the value of $B_z/B_\varphi = 3.8$. Finally, for the known parameters $g = 14$, $m/m_0 = 0.08$ and $\varepsilon_F\tau/\hbar = 0.85$, we obtain from Eq. (A.1): $kT\tau_\varphi/\hbar \approx 0.65$.

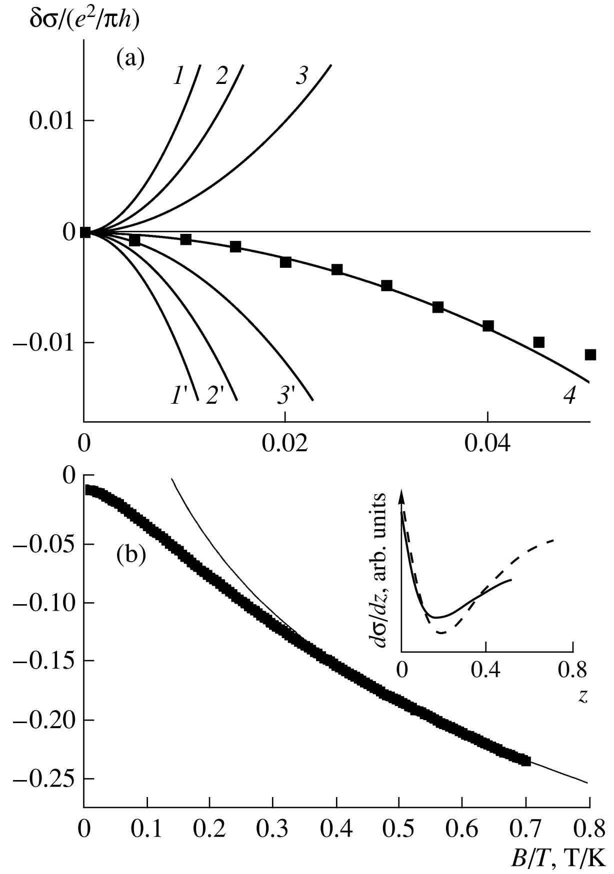

Fig. 3. (a) The best fit of Eq. (8) (solid line *4*) to experimental data for $\delta\sigma$ at $T = 0.4$ K (points) in the weak field limit. Curves *1–3* and *1'–3'* show WL and EEI contributions, respectively, for: $g = 20$, $B_z/B_\varphi = 3.9$ (curves *1*, *1'*); $g = 14$, $B_z/B_\varphi = 3.8$ (curves *2*, *2'*) and $g = 10$, $B_z/B_\varphi = 3.7$ (curves *3*, *3'*). (b) The best fit of Eq. (12) with $\lambda = 0.69$ and $\alpha = 0.7$ (solid line) to experimental data for $\delta\sigma$ at $T = 0.4$ K (points) in the high field limit. Inset shows the fit of theoretical expression (dashed line) to experimental data (solid line) for the first derivative $\partial\sigma(z)/\partial z$ ($z \equiv B/T$) for $g = 14$.

The obtained value of the g-factor is somewhat lower than the theoretical result for $\varepsilon_F \longrightarrow 0$ ($g_\perp = 20.4$); that may be caused by nonparabolicity of the ground hole subband in the Ge quantum well. This result is in accordance with the experimental value of effective mass $m = 0.08m_0$, which, due to nonparabolicity, is higher than the theoretical value $m = 0.056m_0$ on the bottom of the first hole subband in Ge [14a].

The estimation for τ_φ is in rather good accordance with the numerical solution of Eq. (A.2): for $\gamma_2 = 2.2$ and $\varepsilon_F\tau/\hbar = 0.85$ we have $x = 1.3$. The main result of fitting is that we obtain the right order of magnitudes for both the g-factor and τ_φ.

The value of prefactor α may be estimated from high field data, as according to Eqs. (8) and (10) at $B \gg B_z$ [22, 23], $B \gg B_\varphi$ [20], the expression for $\delta\sigma$ reduces to

$$\frac{2\pi^2\hbar}{e^2}\delta\sigma(B) = \text{const} + (\alpha - 2\lambda)\ln\frac{B}{T}. \qquad (12)$$

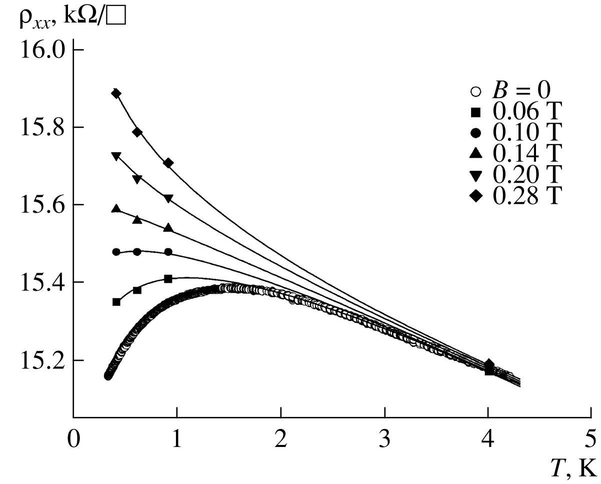

Fig. 4. Magnetoresistivity as a function of temperature in different magnetic fields perpendicular to the 2D-plane.

From the fit of Eq. (12) to experimental data in the high-field limit (see Fig. 3b) we obtain $\alpha = 0.7$, which is in rather good accordance with the theoretical value (9) for our $\sigma = 1.7e^2/h$: $\alpha = 0.63$.

2.2. Magnetic Field Induced Metal–Insulator Transition

In Fig. 4 is shown the resistivity of the investigated sample as a function of temperature at several fixed magnetic fields *perpendicular* to the 2D-plane between 0 and 0.3 T. It is seen that the effect of the magnetic field is mainly observed for $T < T_{max}$ where the conducting ("metallic") phase to insulating phase transition takes place at $B \approx 0.1$ T. We believe that the transition is induced by Zeeman splitting in the electron spectrum that leads to effective suppression of the antilocalizing triplet channel in favor of the localizing exchange channel in the total interaction correction $\delta\sigma_{ee}$ [21–23].

Suppression of the low-temperature conducting phase by a magnetic field $B_\parallel$ *parallel* to the 2D-plane has been first observed in high-mobility Si-MOSFET for electron densities near the zero-field conductor-insulator transition [11, 27]. Also, $B_\parallel/T$ scaling of the magnetoconductance has been found [12], and such behavior attributed just to the Zeeman splitting, with $\delta\sigma_z(B_\parallel, T)$ dependence being fitted to the form suggested in [22] [see Eq. (11)] for $\gamma_2 \approx 1.3$. The effect of Zeeman splitting on the in-plane magnetoconductivity of high-mobility Si-MOSFET in the ballistic regime has been investigated by Pudalov et al. [8] and Vitkalov et al. [28].

In an electron system, the Zeeman splitting suppresses a conducting phase independently of the angle between the field and the 2D-plane (see, for example, [27]). But for a *hole* system with highly anisotropic g-factor ($g_\parallel \ll g_\perp$) this effect in a parallel magnetic field should be weakened considerably. In the magnetic field *perpendicular* to the 2D-plane, we investigate a situation where a WL effect should be present equally with interaction correction, and the usual negative WL magnetoresistance should be observed.

Magnetoresistance in *perpendicular* fields for *p*-SiGe samples on the metallic side of the $B = 0$ metal–insulator transition has been investigated by Coleridge et al. [13]. Magnetoresistance shows clear evidence of both quantum interference and Zeeman interaction effects. The initial NMR attributed to dephasing by the magnetic field due to the WL term is followed by a PMR due to the term identified with the Zeeman interaction effects. The Zeeman term, which scales as B/T, could not be quantitatively described by a conventional theory for a weakly interacting 2D-system [21]. The best fit to the data has been obtained using low- and high-field limits of Castellani et al. Theory [22], with the value of γ_2 up to 2.6.

In a recent work of Gao et al. [29] on the *p*-GaAs system that is metallic at $T \le 0.3$ K, only a negative low-field MR in perpendicular fields shows up, so that the WL effect overwhelms the effect of Zeeman splitting observed in *p*-GaAs in parallel magnetic field [30, 31].

In contrast to the electron Si-MOSFET system [27] or hole SiGe [13] and GaAs [29] systems, we do not observe the low-field NMR and believe that this is a consequence of a particular parameter relation characterizing the investigated *p*-Ge quantum wells, specifically, of a large value of the hole band *g*-factor. In our systems, mechanisms responsible for both (NMR due to WL and PMR due to Zeeman splitting) coexist in the same range of magnetic fields, the latter being predominant.

2.3. Nonmonotonous Temperature Dependence of Resistivity at B = 0

There exist at least two approaches in explanation of the nonmonotonous temperature dependence of resistivity in samples with a low temperature "metallic" phase: for pure diffusive [9] and pure ballistic [10] regimes (see review papers [5]).

In the paper of Punnoose and Finkelstein [9], the highly nonmonotonous temperature dependence of resistivity in a Si-MOSFET sample close to the critical region of the metal–insulator transition [$\rho(T_{max}) \le h/e^2$] has been well described on the basis of renormalization group (RG) theory [15, 22]. According to [15, 22], the interplay of EEI and disorder leads to such a renormalization of the Fermi-liquid parameter γ_2 that it increases monotonically as the temperature is lowered. When γ_2 increases beyond the value γ_2^*, for which $(p + 1 - 3\lambda) = 0$ (see Eq. (7)), the resistivity passes through a maximum.

On the other hand, Das Sarma and Hwang [10] have explained a transition from "metallic" to apparent "insulating" phase with increasing T on the basis of a quasiclassical theory of the temperature dependent screening of the impurity potential [32, 33], suggesting that the resistivity maximum is due to a crossover between the Fermi and Boltzmann statistics. In [6, 7] it is argued that this approach has a common physical origin with the EEI effect at $kT\tau/\hbar \gg 1$, i.e., in a limit of single-impurity scattering, and that the theory of EEI correction in a ballistic regime provides a systematic microscopic view of the concept of temperature dependent screening (see Section IIIF of [6] and Section IV of [7]).

We believe that in our experiment the crossover between diffusive and ballistic regimes with an assumption of smooth character of random impurity potential may be responsible for the nonmonotonous $\rho(T)$ dependence. Really, in a smooth disorder (small angle scattering), the EEI contribution in the ballistic regime, which is proportional to the return probability after a single-scattering event, vanishes as $\exp(-k_F d)$, with d being a spatial range of random impurity potential [7]. As shown in [6, 7], the crossover between diffusive and ballistic limits should take place at small values of $kT\tau/\hbar \approx 0.1$, since the natural dimensionless variable of the theory is $2\pi kT\tau/\hbar$. A flat region in $\rho(B)$ dependencies of a high-mobility *n*-GaAs heterostructure at $T \gtrsim 1.2$ K in fields $\omega_c\tau < 1$ (after the initial rapid drop of MR due to WL dephasing) is interpreted in [34] as a clear indication that in the ballistic regime the long-range potential suppresses the zero-field interaction correction.

For a range of impurity potentials in our structures we have $k_F d \approx 1$, where d is an effective spacer width [25, 35]; thus the impurity potential is of the long-range nature. So, we believe that a transition to the "insulating" behavior at $T > T_{max}$ (see Fig. 1a) takes place due to a gradual change of regime to the ballistic one where the EEI correction at $B = 0$ is exponentially small [7]: $\delta\sigma_{ee} \propto \exp[-\text{const}(T\tau)^{1/2}]$ and thus the WL effect becomes predominant.

Note that for *point-like* scatterers, the linear-in-T contribution (2), (3)

$$\delta\sigma = \frac{e^2}{2\pi^2\hbar}\left(1 + \frac{3F_0^\sigma}{1 + F_0^\sigma}\right)\frac{kT\tau}{\hbar} \equiv \frac{e^2}{2\pi^2\hbar}(1 - 3\gamma_2)\frac{kT\tau}{\hbar}$$

will come to light in the total $\delta\sigma_{ee}$ correction with a transition to the ballistic regime [6]. For our value of γ_2, we have $(1 - 3\gamma_2) \approx -5.6$ and this contribution should lead to a steeper increase of $\rho(T)$ ("more metallic" behavior) with T increasing, which obviously is not the case in our experiment.

3. CONCLUSIONS

We think that a mutual compensation of WL and EEI effects takes place for the investigated *p*-Ge/$Ge_{1-x}Si_x$ heterostructure with parameter values near a nominal 2D "metal–insulator transition", $\rho \approx h/e^2$ ($\varepsilon_F\tau/\hbar \approx 1$). In the pure diffusive regime $kT\tau/\hbar < 0.025$ ($T < 1$ K), the predominance of the antilocalizing triplet channel contribution into EEI correction leads to an apparent metallic

Some experimental data for the Fermi-liquid interaction parameter in diffusive regime

Semiconductor	$\varepsilon_F\tau/\hbar$	r_s	γ_2	F_0^σ	Reference
Si-MOSFET	1.25	–	3.5	–0.78	[36]
Si-MOSFET	1.3	1.6	3.2	–0.76	[37]
p-SiGe	7.2	4	2.6	–0.72	[13]
p-Ge	0.85	1.75	2.2	–0.69	This work
Si-MOSFET	0.93	5.6	1.3	–0.56	[12]

behavior, $\partial\rho/\partial T > 0$. But with a crossover to the ballistic regime (at $kT\tau/\hbar \approx 0.1$), the gradual reduction of the EEI contribution in favor of the WL one ($\partial\rho/\partial T$) < 0 occurs for a smooth or predominantly smooth disorder.

Due to the high value of the Ge valence band *g*-factor, the Zeeman splitting in a magnetic field perpendicular to the 2D-plane causes an effective suppression of the triplet channel contribution and conduces to the insulating behavior of $\rho(T)$ in the whole temperature interval.

Finally, we compare our results for the Fermi-liquid amplitude $\gamma_2(F_2^\sigma)$ with those obtained from an analysis of experimental data on other semiconductor systems in *diffusive* regimes. The highest values of the parameter γ_2 reported for low-mobility [36, 37] and high-mobility [12] Si-MOSFET, as well as for *p*-SiGe [13] and *p*-Ge (this work), are presented in the table. Here $r_s = \pi n^{-1/2}/a_B$ is the usual dimensionless Wigner–Seitz interaction parameter with n as the density of carriers and a_B as the effective semiconductor Bohr radius. Note that all the values of F_0^σ shown in table are appreciably larger (for similar values of r_s) than those obtained from an analysis of transport effects in terms of recent EEI theories in the ballistic regime (see [38]).

It is also seen that almost all of the data in the table (with the exception of the result of [13]) correspond to a region of nominal metal–insulator transition with $\varepsilon_F\tau/\hbar \approx 1$. Then it may be that such a high γ_2 value is a consequence of the renormalization of the Fermi-liquid parameter due to an interplay of interaction and disorder in the diffusive regime, which, according to RG theory [9, 15, 22], is especially significant just in the proximity of $\rho = h/e^2$, i.e., for $\varepsilon_F\tau/\hbar \approx 1$. On the other hand, an apparent reduction of the interaction amplitude extracted from the temperature dependence of resistivity in the ballistic regime may be related to a mixed (point-like plus smooth) character of the random impurity potential (see Eq. (2.53) of [7]).

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APPENDIX A

From the definitions of B_z and B_φ we have

$$\frac{B_z}{B_\varphi} = \frac{4eD}{g\mu_B c}x = \frac{8\varepsilon_F\tau/\hbar}{g(m/m_0)}x, \quad \text{(A.1)}$$

where $x = kT\tau_\varphi/\hbar$ and the expressions $\mu_B = e\hbar/2m_0c$ and $D = \varepsilon_F\tau/m$ are used. Dephasing time in disordered 2D-system due to EEI in diffusive limit is determined by the self-consistent equation [2, 19]

$$\Lambda x \ln x = 2\varepsilon_F\tau/\hbar, \quad \text{(A.2)}$$

with $\Lambda = 1$ for weak EEI ($\gamma_2 \ll 1$) [2] and

$$\Lambda = 1 + \frac{3\gamma_2^2}{\gamma_2 + 2}$$

at an arbitrary value of γ_2 [19] [considering the relation $F_0^\sigma = -\gamma_2/(1+\gamma_2)$]. The solution of Eq. (A.2) may be written as

$$x = f(\gamma_2, \varepsilon_F\tau/\hbar), \quad \text{(A.3)}$$

and for a ratio B_z/B_φ we then have

$$\frac{B_z}{B_\varphi} = \frac{8\varepsilon_F\tau/\hbar}{g(m/m_0)}f(\gamma_2, \varepsilon_F\tau/\hbar), \quad \text{(A.4)}$$

where the right hand part does not depend on temperature.

So, the relation between WL and EEI contributions to the total magnetoconductivity, and thus the sign of MR, should remain constant during the variation of temperature, at least in the weak field region.

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