Quantum Hall effect in p-Ge/Ge$_{1-x}$Si$_x$ heterostructures with low hole mobility

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The apparent insulator–quantum Hall–insulator (I–QH–I) transition for filling factor $\nu=1$ is investigated in p-type Ge/Ge$_{1-x}$Si$_x$ heterostructures with $\varepsilon_F\tau/hbar\approx 1$. Scaling analysis is carried out for both the low- and high-field transition point. In low magnetic fields $\omega_c\tau<1$, pronounced QH-like peculiarities for $\nu=1$ are also observed in both the longitudinal and Hall resistivities. Such behavior may be evidence of a localization effect in the mixing region of Landau levels and is inherent to two-dimensional structures in the vicinity of a metal–insulator transition.
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INTRODUCTION

A magnetic-field-induced transition from an Anderson insulator to quantum Hall effect (QHE) conductor has been reportedly observed both for low-electron-mobility GaAs/AlGaAs heterostructures[1–4] and low-hole-mobility Ge/SiGe quantum wells,[5,6] which at magnetic field $B=0$ exhibit insulating behavior with a divergent resistance $\rho(T\to 0)\gg h/e^2$. An initial very large decrease of diagonal resistivity ρ_{xx} (giant negative magnetoresistence[7]) is followed by a clear critical point at $B=B_C$, where the ρ_{xx} value is temperature independent. At higher fields the QHE minima for filling factor either $\nu=2$ or $\nu=1$ are developed. The insulator-to-QHE boundary points at $B=B_C$ are characterized by equality of the diagonal and Hall resistivities, $\rho_{xxc}=\rho_{xyc}$, within the experimental uncertainty.[5] It is the T-independent point B_C that was identified by the authors of Refs. 1–6 as the quantum phase transition point between the insulator and QHE conductor.

In contrast to that, Huckestein[8] identifies the apparent low-field insulator-QHE transition as a crossover due to weak localization and a strong reduction of the conductivity when Landau quantization becomes dominant at $\omega_c\tau\geq 1$ (ω_c is the cyclotron frequency and τ the elastic mean free time).

On the other hand, for well-conducting 2D systems with $k_Fl\gg 1$ (k_F is the Fermi quasimomentum and l is the mean free path) the interplay of classical cyclotron motion and the quantum correction $\Delta\sigma ee$ to the Drude conductivity $\sigma_D=(e^2/h)(k_Fl)$ due to electron-electron interaction (EEI) leads to a parabolic negative magnetoresistance:[9–11]

$$\rho_{xx}(B,T)=\frac{1}{\sigma_D}+[1-(\omega_c\tau)^2]\frac{\Delta\sigma_{ee}(T)}{\sigma_D^2}. \qquad (1)$$

The temperature-independent point at $\omega_c\tau=1$ (for $\rho_{xx}\cong\rho_{xy}$) predicted by Eq. (1) has been observed in various experiments and used for the estimation of the σ_D value (see, for example, Refs. 12–15).

It seems to us that the results of Huang *et al.*[16] are an especially beautiful experimental demonstration of just this (EEI) physical picture in a gated GaAs/AlGaAs heterostructure (our estimates give $4\leq k_Fl\leq 13$ for five V_g values in their Fig. 2), but the authors of Ref. 16 treated the low-field T-independent point as a kind of quantum phase transition (see also Ref. 17).

Here we report and analyze the results of magnetotransport measurements for low-mobility p-Ge/Ge$_{1-x}$Si$_x$ heterostructures, where the low-field temperature-independent point on the $\rho_{xx}(B)$ curve is clearly observed.

EXPERIMENTAL RESULTS AND DISCUSSION

Experimental data are presented for two samples A and B of a multilayered Ge/Ge$_{1-x}$Si$_x$ p-type heterostructures. The hole density and Hall mobility, as obtained from the zero-field resistivity ρ_0 and low-field Hall coefficient at $T=4.2$ K, are $p=1.3(1.1)\times 10^{11}$ cm^{-2} and $\mu=3.6(4.0)\times 10^3$ cm^2(V·s) ($\rho_0=16(15)$ kΩ/□). From the relation $\rho_0^{-1}=(e^2/\pi\hbar)(\varepsilon_F\tau/\hbar)$ the important parameter connecting the Fermi energy ε_F and elastic mean free time τ may be estimated: $\varepsilon_F\tau/\hbar=0.8(0.85)$. Thus for the samples investigated $\varepsilon_F\tau/\hbar\approx 1$, and we are in a region of conjectural metal–insulator transition, which is seen experimentally in a variety of two-dimensional semiconductor systems.[18]

The dependences of the longitudinal ρ_{xx} and Hall ρ_{xy} resistivities on magnetic field B at $T=1.7-4.2$ K up to $B=12$ T for sample A are shown in Fig. 1. The quantum Hall effect (QHE) plateau number one and the corresponding ρ_{xx} minimum at $B\approx 35$ T are well seen in the figures. An estimate of the hole mobility from the condition $\mu B_{C1}=1$, where B_{C1} (=2.7 T) is the field at which $\rho_{xx}=\rho_{xy}$ (see Fig. 1a), gives

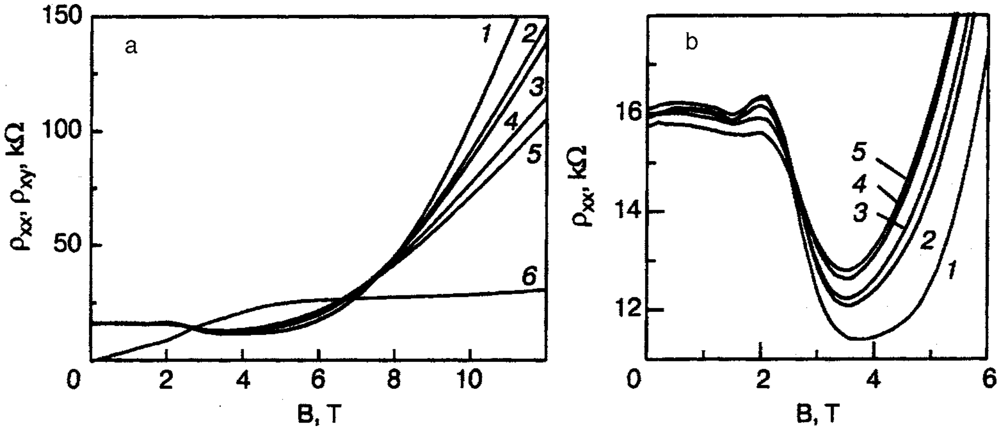

FIG. 1. Longitudinal resistivity (*1–5*) and Hall resistivity (*6*) as functions of magnetic field for sample A. T [K]: *1,6*—1.7; *2*—2.3; *3*—2.9; *4*—3.7; *5*—4.2.

$\mu=3.7\times10^3$ cm^2(V·s), in reasonable accordance with the low-field estimate.

We take notice that at $B<0.5$ T, positive magnetoresistance due to the effect of Zeeman splitting[19] is observed for all temperatures. At fields $B>0.5$ T up to the QHE minimum of ρ_{xx} a background negative magnetoresistance takes place, and the following peculiarities are observed: i) Shubnikov-de Haas (SdH) oscillation structure with maximum at $B\approx2$ T, and ii) a temperature-independent point of ρ_{xx} at $B\approx B_{C1}$ (Fig. 1b). In the high-field region the transition from the QHE regime to the insulator takes place in the vicinity of $B_{C2}\approx7.5$ T (Fig. 1a).

In many studies[1–6,16,17] the low-field temperature-independent point at $B=B_C$ on the $\rho_{xx}(B)$ curve has been interpreted as the point of an insulator-QHE quantum phase transition. A criterion of existence of a phase transition is scaling dependence of $\rho_{xx}(B,T)=f((B-B_C)/T^{\kappa})$ in the vicinity of B_C (κ is a critical exponent).[20] By plotting $\ln(d\rho_{xx}/dB)_{B=B_C}$ versus $\ln T$, one could obtain κ. Such a situation may be realized in a system with genuine (strong) localization, e.g., with variable range hopping conduction at $B=0$.

For a system with weak localization, however, we think that that is not the case. The weak localization regime at $k_F l\gg1$ ($\varepsilon_F\tau/\hbar\gg1$) is in fact the regime of electron diffusion from one scattering event on an impurity to another, with some mean free path l. Here the notion of insulating behavior is valid only in the sense that $d\rho/dT<0$. For such a system there exists another reason for a temperature-independent point on the $\rho_{xx}(B)$ curve at $\omega_c\tau=1$ ($B_{C1}=mc/e\tau$): it is a consequence of the interplay of classical cyclotron motion and the EEI correction $\Delta\sigma_{ee}$ to the Drude conductivity (see Eq. (1)). According to Eq. (1) the derivative $(d\rho/dT)_{B=B_C}$ should be proportional to $\ln T$, as $\Delta\sigma_{ee}$ is proportional to $\ln(kT\tau/\hbar)$.

To distinguish between the two cases in our samples with $\varepsilon_F\tau/\hbar\cong1$ an analysis of the dependence of $(d\rho_{xx}/dB)_{B=B_C}$ on T was carried out. Figure 2a shows the nonscaling behavior of $\rho_{xx}(B,T)$ near the low-field critical point B_{C1}: it is not possible to extract consistently any power law from the temperature dependence of the derivative $(d\rho_{xx}/dB)_{B=B_{C1}}$. On the other hand, rather good linear dependence of $(d\rho_{xx}/dB)_{B=B_{C1}}$ on $\ln T$ is observed up to $T\approx3$ K, which is an argument in favor of the EEI version. In contrast to it, real scaling behavior of $\rho_{xx}(B,T)$ with critical exponent $\kappa=0.38$ (compare with the theoretical value $\kappa=0.42$ for the spin-split case[21]) takes place in the vicinity of the high-field critical point B_{C2} (Fig. 3).

The experimental data for sample B at $T=0.4$ K are presented in Fig. 4. The QHE plateau number one and the corresponding minimum at $B=5.6$ T are clearly seen on the $\rho_{xy}(B)$ and $\rho_{xx}(B)$ curves. Estimation of the hole mobility

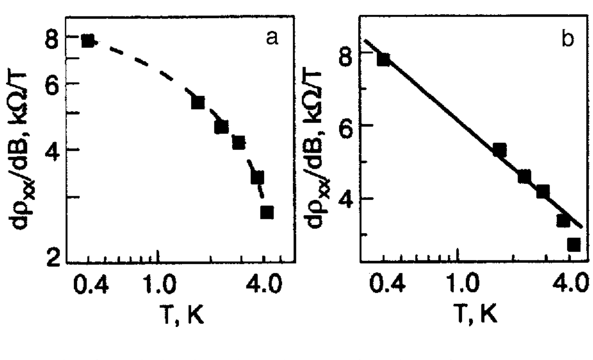

FIG. 2. The first derivative $d\rho_{xx}/dB$ as a function of temperature in the vicinity of the low-field critical point in log-log scale (a) and linear-log scale (b). The dashed line in panel "a" is a guide to the eye.

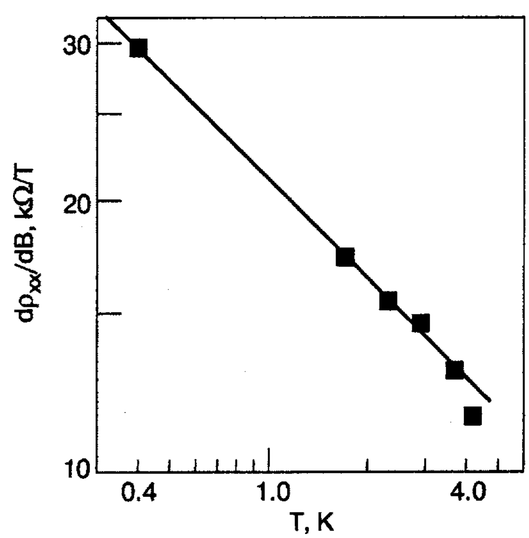

FIG. 3. The first derivative $d\rho_{xx}/dB$ as a function of temperature in the vicinity of the high-field critical point (log-log scale).

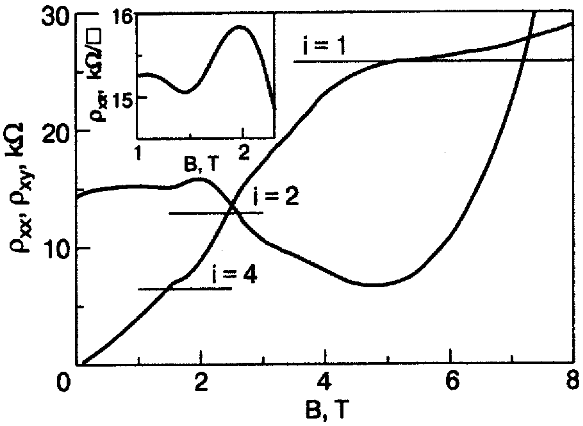

FIG. 4. Longitudinal and Hall resistivities as functions of magnetic field for sample B at $T=0.4$ K.

from the $\rho_{xx}=\rho_{xy}$ point $B_{C1}=2.5$ T gives $\mu=4.0\times 10^3$ cm^2/(V·s). The condition for the fields of the QHE minima of $\rho_{xx}(B)$, $p=i(e/hc)B_i$, where i is the number of the plateau, gives $p=1.2\times 10^{11}$ cm^{-2}.

It is seen from Fig. 4 that in the low-field region $B<B_{C1}$ ($\omega_c\tau\approx 0.7$) the minimum in $\rho_{xx}(B)$ at $B_4=1.4$ T (see the inset of this figure) and the precursor of $\rho_{xy}(B)$ plateau number four are observed. Really, Fig. 5 shows pronounced QHE-like structures on the dependence of the first derivative $d\rho_{xy}/dB$ on filling factor for $\nu=1$, 2, and 4.

In the complete QHE regime at $\omega_c\tau\gg 1$ the appearance of quantized plateaus on the $\rho_{xy}(B)$ curves with vanishing values of ρ_{xx} is commonly held to be caused by the existence of disorder-induced mobility gaps (stripes of localized states) between the narrow bands of extended states of width Γ presented close to the center of each of the Landau subbands.[22] The existence of QHE-like structures at $\omega_c\tau<1$ should then be a manifestation of localization of electron states in the mixing regions for adjacent Landau subbands, so that the width of extended state bands is less than the collision broadening of the Landau level: $\Gamma<\hbar/\tau$. We think that the realization of such a situation is more likely just for $\varepsilon_F\tau/\hbar\cong 1$, when the localization effect is more substantial than for $\varepsilon_F\tau/\hbar\gg 1$ but is not yet too strong, as for $\varepsilon_F\tau/\hbar\ll 1$.

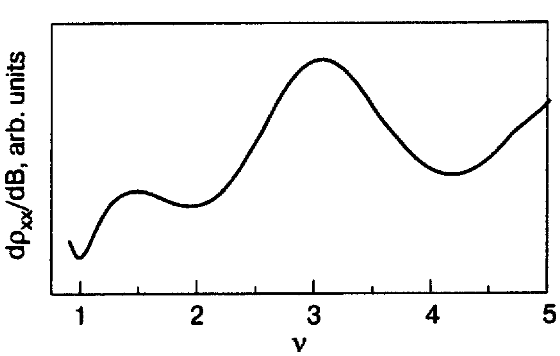

FIG. 5. The first derivative $d\rho_{xy}/dB$ as a function of the filling factor ν for sample B at $T=0.4$ K.

CONCLUSIONS

Both low-field (B_{C1}) and high-field (B_{C2}) T-independent points on the $\rho_{xx}(B)$ curve, with the $\nu=1$ QHE state between them, have been observed for p-type Ge/Ge$_{1-x}$Si$_x$ heterostructures with low hole mobility ($k_Fl\approx 1.6$). In contrast to the series of works[1–6,16,17] in which the low-field point is treated as the critical point of an insulator→QHE phase transition, we speculate that in our 2D systems with $k_Fl\geq 1$, such a point at $\omega_c\tau=1$ is a manifestation of a quantum e-e interaction correction in the diagonal component of the magnetoresistivity tensor.

On the other hand, in accordance with Refs. 1–6 the high-field B_{C2} point is a point of genuine quantum phase transition between the $\nu=1$ QHE phase and the high-field insulator and corresponds to the passing of the Fermi level through the lowest Landau level.

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