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Galvanomagnetic Study of the Quantum-Well Valence Band of Germanium in the $Ge_{1-x}Si_x$/Ge/$Ge_{1-x}Si_x$ Potential Well

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Abstract—The structure of the quantum-well valence band in a Ge(111) two-dimensional layer is calculated by the self-consistent method. It is shown that the effective mass characterizing the motion of holes along the germanium layer is almost one order of magnitude smaller than the mass for the motion of heavy holes along the [111] direction in a bulk material (this mass is responsible for the formation of quantum-well levels). This creates a unique situation in which a large number of subbands appear to be populated at moderate values of the layer thickness d_w and the hole concentration p_s. The depopulation of two or more upper subbands in a 38-nm-thick germanium layer at a hole concentration $p_s = 5 \times 10^{15}$ m^{-2} is revealed from the results of measuring the magnetoresistance in a strong magnetic field aligned parallel to the germanium layers. The destruction of the quantum Hall state at a filling factor $\nu = 1$ indicates that the two lower subbands merge together in a self-formed potential profile of the double quantum well. It is demonstrated that, in a quasi-two-dimensional hole gas, the latter effect should be sensitive to the layer strain.

1. INTRODUCTION

Upon addition of one more degree of freedom to the two-dimensional motion of charge carriers in a layer, there arise favorable conditions for the occurrence of new physical phenomena in semiconductor heterosystems. For example, these conditions can either bring about the formation of new electronic phases in a multicomponent system residing in the quantum Hall state or extend the range of existence of the phases formed in a two-dimensional layer [1].

Such a crossover can be most efficiently accomplished by the following two methods. The first method consists in providing the possibility of populating upper quantum-well subbands. In particular, Sergio *et al.* [2] demonstrated that the population of at least eight subbands can be achieved in a wide parabolic potential well filled with electrons in which two- and three-dimensional states coexist. Gusev *et al.* [3] revealed a new collective state. It was found that collective states in a hole gas are more readily generated than those in an electron gas due to the greater mass of the holes [4].

The second method involves the fabrication of a system composed of interrelated two-dimensional layers in which new phenomena of the physics of multicomponent systems can manifest themselves owing to the formation of interlayer correlated states [1, 5]. It should be noted that the hole systems are more promising due to the greater mass of the holes, because, in this case, the interlayer tunneling that hinders the formation of interlayer correlated states is suppressed [6].

In this work, we investigated the magnetotransport phenomena in a quasi-two-dimensional hole gas in a germanium layer under conditions where a large number of subbands are populated. The system under consideration was doped selectively. This brought about a bending of the potential well bottom and the formation of a double-quantum-well profile. In turn, this resulted in separation of the hole gas into two two-dimensional sublayers in germanium layers of large thickness.

2. OBJECTS OF INVESTIGATION

The measurements were performed for a series of $Ge_{1-x}Si_x$/Ge/$Ge_{1-x}Si_x$ ($x \approx 0.1$) quantum wells grown on a (111) substrate. The central region of the $Ge_{1-x}Si_x$ barriers was doped with boron. The samples to be studied had different thicknesses d_w of the germanium layer and different hole gas densities p_s in this layer. The parameters of the samples are given in the table.

The quantum Hall effect was observed when the magnetic field was applied perpendicularly to the ger-

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Parameters of the studied samples

Sample no.	d_w, nm	p_s, 10^{15} m^{-2}
578	8	1.4
1006	12.5	4.9
1123	23	3.4
1124, 1125	22	2.8
475a2	38	5
476b4	38	5.8

manium layers (Fig. 1). This effect was thoroughly analyzed in a number of our previous works (see, for example, [7]). This paper reports on the results of investigations performed for thicker layers and data on the magnetoresistance of the same samples in a magnetic field aligned parallel to the layers.

3. RESULTS AND DISCUSSION

The specific features of the quantum Hall effect at the filling factor $\nu = 1$ (i.e., the plateau at the Hall resistance $R_{xy} = 25.8$ kΩ and the corresponding minimum of the longitudinal magnetoresistance ρ_{xx}) are not observed for germanium layers with a thickness of greater than ~30 nm and are well pronounced for thinner layers. This behavior is associated with the formation of a double quantum well (see below).

The results of the measurements in a parallel magnetic field are presented in Fig. 2. For convenience of comparative analysis, the results obtained for different samples are normalized to the resistance ρ_0 in a zero magnetic field.

The parallel magnetic field has virtually no effect on the magnetoresistance of germanium layers with the smallest thickness (see the data in Fig. 2 for sample 578 with a layer thickness $d_w \approx 8$ nm). The results of the calculations (Fig. 3) clearly demonstrate that only one hole quantum-well subband is populated in this sample. Samples with thicker germanium layers possess a strong negative magnetoresistance (up to 40% of the zero-field resistance ρ_0). It is worth noting that the negative magnetoresistance of germanium layers with a moderate thickness ($d_w \approx 20$ nm) is described by a smooth curve, whereas the negative magnetoresistance of layers with the largest thickness (~40 nm; samples 475a2, 476b4) is characterized by a monotonic curve with local features [8]. These features are clearly seen after subtracting the monotonic background, which was simulated by a fourth-degree polynomial (see inset to Fig. 2). The experiments performed in tilted magnetic fields showed that the above features manifest themselves in a narrow range of tilt angles in the vicinity of the magnetic field aligned parallel to the germanium layers when Shubnikov–de Haas oscillations have already disappeared.

The quantum-well structure of the valence band in the Ge(111) layer was calculated by self-consistently solving a system of Schrödinger equations (on the basis of the Luttinger Hamiltonian with due regard for the exchange–correlation energy [9]) and Poisson equations. The main features of the calculated valence band structure can be summarized as follows (Figs. 3, 4).

(1) The subbands in the Ge(111) layer have a relatively simple structure. Although the shape of the subbands differs significantly from parabolic, they do not contain additional extrema. This is inconsistent both with the predictions made for an infinitely deep poten-

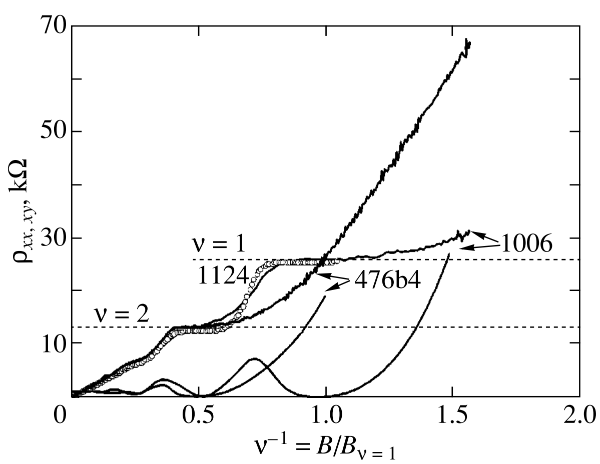

Fig. 1. Quantum Hall effect in germanium layers with thicknesses $d_w < 30$ nm (samples 1006, 1124) and $d_w > 30$ nm (sample 476b4).

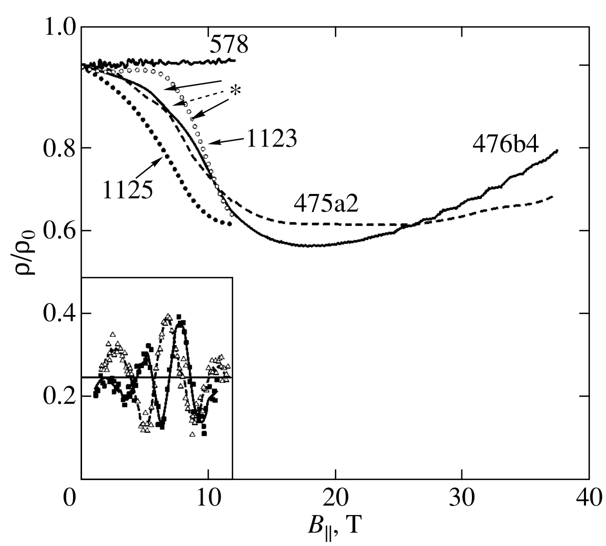

Fig. 2. Magnetoresistances of different samples in a parallel magnetic field at $T = 1.6$ K. The asterisk indicates the local features in the magnetoresistance of samples 475a2 and 476b4. The inset shows the magnetoresistances of samples 475a2 and 476b4 after subtracting the monotonic component.

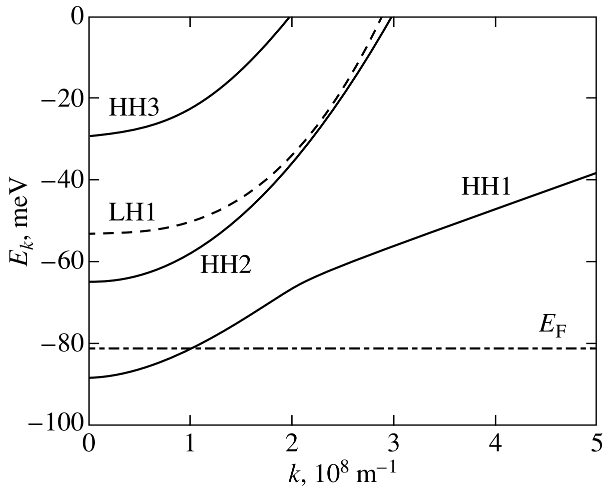

Fig. 3. Structure of the valence band in an 8-nm-thick germanium layer.

tial well [10] and with the results of the valence band calculations performed for a GaAs(100) layer (see, for example, [11]) but agrees with the results of the calculations performed by Winkler *et al.* [12] for Ge(100) layers.

(2) The energy dispersion $E_i(k_{\in\langle 111\rangle})$ for the extreme hole subband is characterized by a rather small effective mass m/m_0 = 0.053–0.062, which is close to the light-hole mass $m_{LH\langle 111\rangle}/m_0$ = 0.040 in bulk germanium [13]. The small effective mass is due to a considerable mixing of heavy- and light-hole bulk states in the wave function of the subband at $k_\parallel \neq 0$, whereas the states in the extreme subband at $k_\parallel = 0$ correspond to heavy holes. Herein lies a radical difference between the valence and conduction bands. Actually, in the conduction band, the character of the wave functions in the subband remains almost unchanged even though $k_\parallel$ is varied, whereas the mass of electrons in the subbands is equal to the bulk mass and increases with an increase in $k_\parallel$, thus reflecting only an insignificant nonparabolicity due to the influence of the nearest bands. A combination of small masses of the holes for motion along the layer with large bulk masses of the heavy germanium holes, $m_{HH\langle 111\rangle}/m_0$ = 0.50 [13] (this mass is responsible for the formation of quantum-well levels), creates a unique situation where a large number of subbands are populated at moderate values of the hole concentration and the layer thickness.

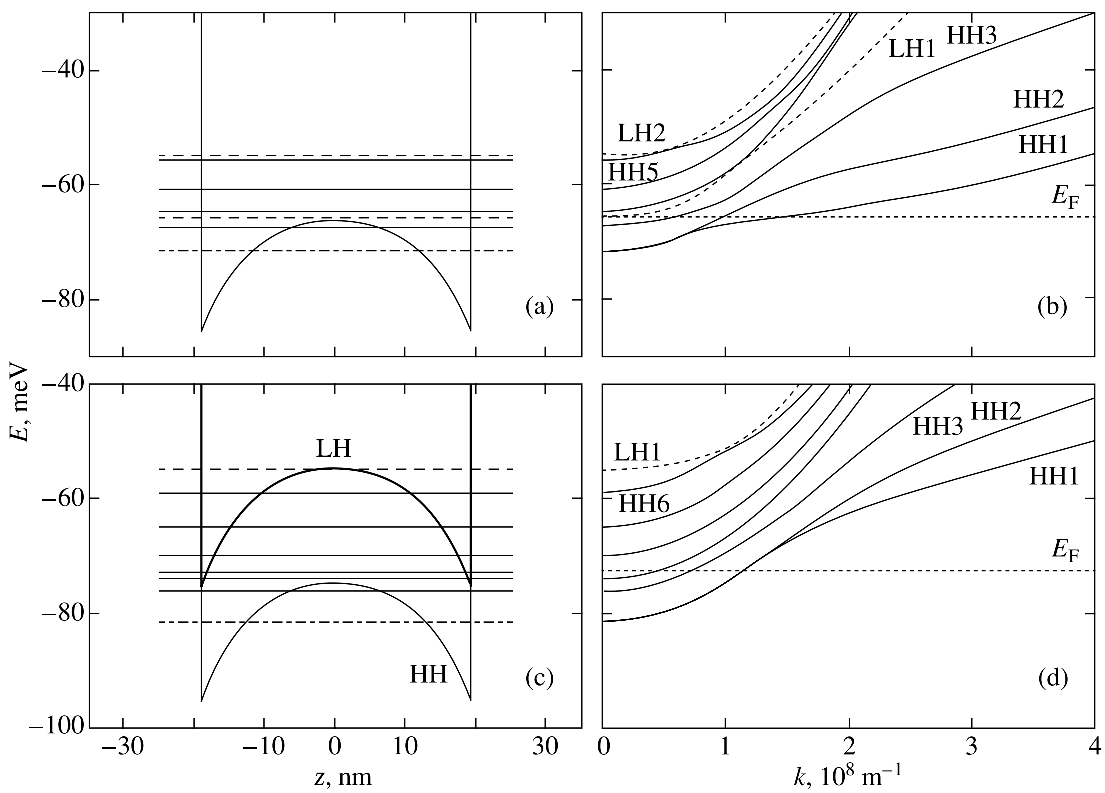

Fig. 4. Calculated valence band structure for sample 475a2. The energy increases deep into the valence band. The potential profiles and energy levels are calculated (a) without regard for the strain and (c) at the strain parameter ζ = 10 meV. The structures of the corresponding subbands and the Fermi levels are calculated (b) without regard for the strain and (d) at the strain parameter ζ = 10 meV.

According to the performed calculations, the fact that the specific features revealed in the quantum Hall effect disappear at the filling factor $\nu = 1$ in thick germanium layers can be explained by merging the two extreme subbands HH1 and HH2 in the self-formed double-quantum-well potential (Fig. 4a), but only in the case where the layer strain is taken into account. It can be seen from Figs. 4b and 4d that the two extreme subbands merge together only at small values of $k_{\parallel}$ and diverge from each other with an increase in $k_{\parallel}$. This is yet another specific feature in the quantum-well spectrum of the degenerate valence band. If two levels of the electron gas in the double quantum well were to coincide with each other at $k_{\parallel} = 0$, their subbands would remain virtually merged with an increase in $k_{\parallel}$ over the entire range of energies. In an unstrained germanium layer at energies in the vicinity of the Fermi level, these subbands are rather widely separated (Fig. 4b) and the gap at the Fermi level should bring about the formation of a quantum Hall state at the filling factor $\nu = 1$. However, the layer strain extends the range in which the subbands merge together and the subbands at a strain parameter $\zeta = 10$ meV remain merged at the Fermi level (Fig. 4d). Since the gap at the Fermi level is absent, the Landau levels of both subbands (i.e., in two sublayers) coincide in pairs and only even features of the quantum Hall effect should manifest themselves for the germanium layer as a whole (this is actually observed in the experiment).

The above splitting of the hole subbands stems from the fact that the fractions of the states of light holes, which are admixed to levels of symmetric and antisymmetric states, are different at energies close to and above the barrier in the double quantum well. The germanium layers in the $Ge_{1-x}Si_x$ alloy are uniaxially stretched along the growth direction due to the smaller lattice spacing. This leads to a shift of the light-hole subbands deep into the valence band. As a consequence, the nonparabolicity range of heavy-hole subbands is shifted to high energies and the range in which the HH1 and HH2 subbands merge together increases and reaches the Fermi level at $k_{\parallel} = k_F$. Therefore, in the absence of strains in samples 475a2 and 476b4, there should exist a gap of ~2 meV at the Fermi level. At liquid-helium temperatures, this would reliably provide the formation of a quantum Hall state at the filling factor $\nu = 1$. However, for a strain parameter (half the strained gap) $\zeta > {\sim}6$ meV, the range in which the subbands merge together reaches k_F. In this case, the state at the Fermi level appears to be either doubly degenerate or quadruply degenerate with allowance made for the spin. In the magnetic field, the levels remain doubly degenerate after spin splitting and only even quantum Hall states manifest themselves in the experiment. The interlayer correlation effects additionally contribute to the destruction of quantum Hall states at the filling factor $\nu = 1$ [1, 5, 14].

The results of the calculations demonstrate that, apart from the two extreme merged subbands, one or two higher lying subbands are populated in a 38-nm-wide potential well at a hole concentration $p_s \approx 5 \times 10^{15}$ m^{-2} (Fig. 4). We believe that the population of these upper subbands is responsible for the local features observed in the magnetoresistance $\rho(B_{\parallel})$ of samples 475a2 and 476b4. Owing to the upward diamagnetic shift of the subbands, the Fermi level intersects them sequentially and each intersection manifests itself as a feature in the magnetoresistance due to the change in the density of states at the Fermi level and the suppression of intersubband scattering. The observed local features of the magnetoresistance (two, at the minimum) suggest that at least two upper subbands are populated in a zero magnetic field.

It should be noted that samples 1123 and 1124 with germanium layers of moderate thickness (~20 nm) also have a negative magnetoresistance of the same type and magnitude but without local features. This indicates that only one subband is depopulated in the parallel magnetic field. A decrease in the resistance $\rho(B_{\parallel})$ of sample 1123 is observed in magnetic fields stronger than those for sample 1125 (Fig. 1). Since the former sample is characterized by a higher hole concentration and a wider potential well, the above difference can be explained by the fact that, for sample 1123 in a zero magnetic field, the Fermi level is located deeper in this upper subband.

In conclusion, we note that the possibility of populating many subbands in the studied samples suggests that this hole heterosystem is promising for the search for correlated states at ultralow temperatures.

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