# Nonmonotonic temperature dependence of the resistivity of $p$-Ge/$Ge_{1-x}Si_x$ in the region of the metal–insulator transition

Yu. G. Arapov, G. I. Harus, N. G. Shelushinina, and M. V. Yakunin

*Institute of Metal Physics of the Ural Division of the Russian Academy of Sciences, Sofia Kovalevskaya St., Ekaterinburg 620219, Russia*

V. N. Neverov*

*Institute of Metal Physics of the Ural Division of the Russian Academy of Sciences, Sofia Kovalevskaya St., Ekaterinburg 620219, Russia; Ural State University, pr. Lenina 51, Ekaterinburg 620083, Russia*

O. A. Kuznetsov

*Physicotechnical Research Institute, Nizhny Novgorod State University, Nizhny Novgorod 603600, Russia*

L. Ponomarenko and A. De Visser

*Van der Waals-Zeeman Amsterdam University, Valknierstraat 65, 1018 XE Amsterdam, The Netherlands*
(Submitted May 11, 2004)
Fiz. Nizk. Temp. **30**, 1157–1161 (November 2004)

In a two-dimensional (2D) hole system (multilayer $p$-Ge/$Ge_{1-x}Si_x$) heterostructure with conductivity $\sigma \approx e^2/h$ at low temperatures ($T \approx 1.5$ K) a transition from the insulator phase ($d\sigma/dT > 0$) to a "metallic" phase ($d\sigma/dT < 0$) is observed as the temperature is lowered, behavior that is in qualitative agreement with the predictions of the Finkelstein theory. In a magnetic field **B** perpendicular to the plane of the 2D layer one observes positive magnetoresistance depending only on the ratio $B/T$. We attribute the positive magnetoresistance effect to the suppression of the triplet channel of Fermi-liquid electron–electron interaction by the magnetic field owing to the strong Zeeman splitting of the hole energy levels.  [DOI: 10.1063/1.1819865]

## INTRODUCTION

In disordered 2D systems at low temperatures there are two types of quantum corrections $\delta\sigma = \delta\sigma_{wl} + \delta\sigma_{ee}$ to the Drude conductivity $\sigma_0 = e^2/h(k_F l)$: $\delta\sigma_{wl}$ is the correction due to inertial effects in the scattering of the electron waves on impurities (weak localization), and $\delta\sigma_{ee}$ is the correction due to the disorder-modified electron–electron interaction.[1,2] In weakly disordered systems with $k_F l > 1$ these corrections are small in the parameter $(k_F l)^{-1}$ ($l$ is the mean free path) and depend logarithmically on temperature.

Experiments to detect[3] and study (see reviews[4,5]) the so-called metal–insulator transition from the change in the carrier density in 2D semiconductor structures with high mobility have stimulated a substantial advance in the theory of electron–electron interaction effects.[6,7] The general theory of quantum corrections to the components of the conductivity tensor of a 2D system owing to electron–electron interaction effects has been developed for the case $kT < E_F$ for an arbitrary relationship between the values of $kT$ and $\hbar/\tau$ ($\tau$ is the momentum relaxation time) over the whole range of temperatures from the diffusion regime ($kT\tau/\hbar \ll 1$) to the ballistic regime ($kT\tau/\hbar \gg 1$) both for short-range (point)[6] and for large-scale (smooth)[7] impurity potentials.

For example, the linear growth of the resistivity $\rho$ with temperature in Si-MOSFET structures with high carrier mobility at large values $\sigma_0 \gg e^2/h$, which for the past decade has been considered to be a manifestation of an "anomalous metallic" state, is now interpreted as being due to an electron–electron interaction effect in the ballistic regime.[8] However, the nonmonotonic temperature dependence of $\rho(T)$ observed near the proposed metal–insulator transition ($\sigma_0 \approx e^2/h$) still does not have a generally accepted explanation. This has stimulated our investigations into multilayer $p$-Ge/$Ge_{1-x}Si_x$ heterostructures.

Suppression of the low-temperature conducting phase by a magnetic field *parallel* to the 2D layer (*positive* magnetoresistance) has been observed repeatedly for high-mobility Si-MOSFET[9–15] and $p$-GaAs heterostructures[16,17] such behavior is explained either by the "complete polarization" of the electron (hole) gas[12–14,17,18] or (at low fields) by the Zeeman effect in the quantum correction owing to the electron–electron interaction in both the diffusion[19] and ballistic[8,15] regimes.

We have carried out studies in a magnetic field *perpendicular* to the 2D layer, where together with the Zeeman level splitting it is necessary to take weak localization effects into account. The hole gas in the Ge quantum wells for the $p$-Ge/$Ge_{1-x}Si_x$ heterostructures studied is described by the Luttinger Hamiltonian with a highly anisotropic $g$ factor in respect to the mutual orientation of the magnetic field and the 2D plane. At the bottom of the lower spatial subband $g_\perp = 6\kappa$ (where for Ge the Luttinger parameter $\kappa = 3.4$)[20] for the *perpendicular* magnetic field and $g_\parallel = 0$ for the *parallel*.[21,22] For interpretation of our experimental $\rho(B,T)$ curves in the samples near the proposed metal–insulator phase transition we invoked a model used for semiconduct-

 

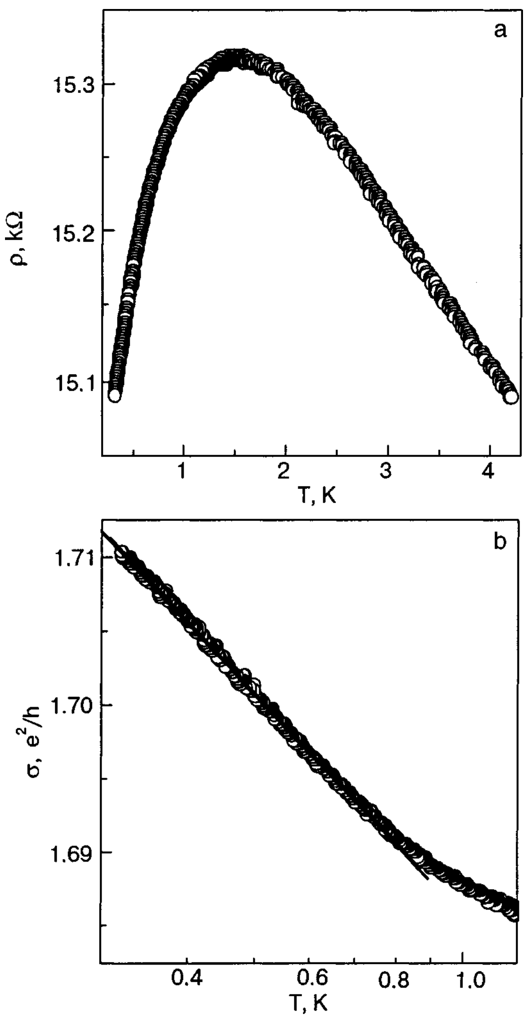


FIG. 1. Temperature dependence of the resistivity (a) and conductivity (b) for $B=0$.

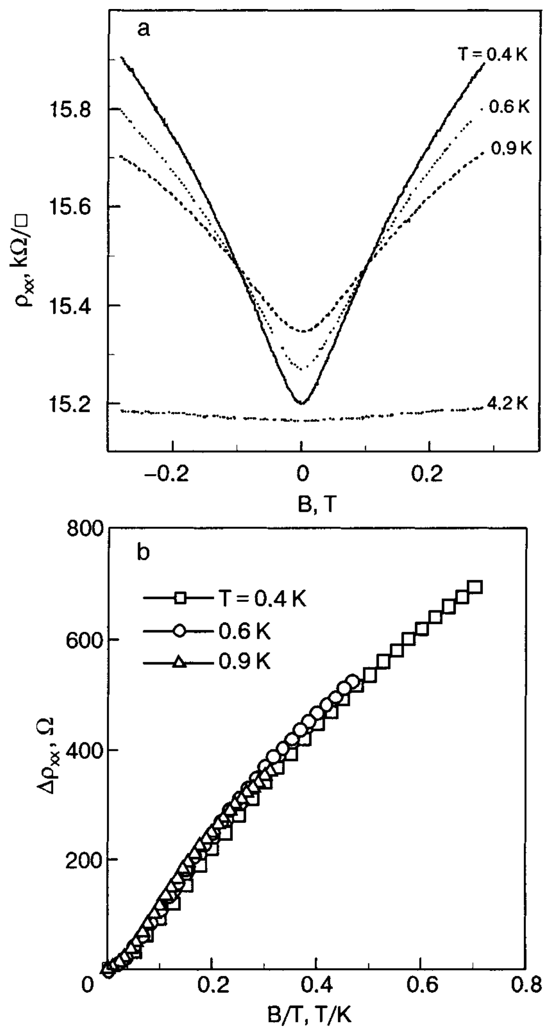


FIG. 2. Resistivity versus magnetic field (a) and magnetoresistivity versus *B/T* (b) at different temperatures.

ing 2D systems with high mobility.[10,15,19,23,24]

## EXPERIMENTAL RESULTS AND DISCUSSION

Measurements of the galvanomagnetic effects in multilayer heterostructures of $p$-type $Ge/Ge_{1-x}Si_x$ were made in magnetic fields up to 5 T at $T=0.3$–4.2 K. For a sample[1)] with a carrier density of $1.2\times10^{11}$ cm$^{-2}$ and mobility $\mu_p=4\times10^3$ cm$^2$/(V·s) (parameter $\varepsilon_F\tau/\hbar=0.75$), nonmonotonic low-temperature behavior of the resistivity is observed (Fig. 1a): $\rho(T)$ increases with decreasing temperature from 4.2 to 1.5 K (localization) and then $\rho(T)$ decreases as $T$ is lowered further from 1.5 to 0.3 K (antilocalization). In the antilocalization region for $T\leqslant 1$ K the conductivity depends logarithmically on temperature (Fig. 1b). In the whole temperature interval positive magnetoresistance is observed, increasing sharply with decreasing $T$ (Fig. 2a). At low temperatures $T<1$ K in fields $B<0.3$ T the magnetoresistivity $\Delta\rho_{xx}$ is an almost universal function of the ratio $B/T$ (Fig. 2b).

The observed $\rho(B,T)$ curves can be compared with the quantum corrections to the two-dimensional conductivity due to the weak localization effects ($\delta\sigma_{wl}$) and to electron–electron interaction ($\delta\sigma_{ee}$). For the electron–electron interaction effects in the diffusion regime $k_BT\tau/\hbar\ll 1$ we have[1,2]

$$\delta\sigma_{ee}(B,T)=\delta\sigma_{ee}(0,T)+\delta\sigma_z(b), \tag{1}$$

where

$$\delta\sigma_{ee}(0,T)=\frac{e^2}{2\pi^2\hbar}(1-3\lambda)\ln\frac{k_BT\tau}{\hbar}, \tag{2}$$

$$\delta\sigma_z(b)=-\frac{e^2}{2\pi^2\hbar}G(b)\quad\left(b=\frac{g\mu_BB}{k_BT}\right). \tag{3}$$

The first term in front of the logarithm in Eq. (2) corresponds to the exchange part of the electron–electron interaction, while the second term corresponds to the Hartree contribution (triplet channel). Here

$$\lambda=\frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2)-1, \tag{4}$$

where $\gamma_2$ is the Fermi-liquid interaction parameter.[25]

The electron–electron contribution of the magnetic field is given as a function of the ratio $B/T$ by expression (3), where $G(b)$ is a known function describing the positive magnetoresistance due to the splitting of the electron energy levels,[1,26,27] and $g=20.4$ for a 2D hole gas in Ge for $\varepsilon_F\to 0$.

For weak localization effects[28]

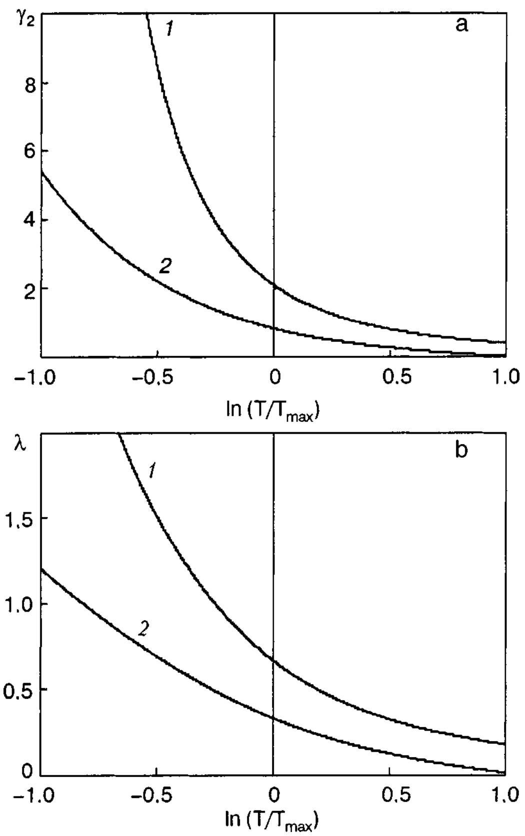


FIG. 3. Temperature dependence of the parameter $\gamma_2$ (a) and $\lambda=[(1+\gamma_2)/\gamma_2]\ln(1+\gamma_2)-1$ (b) according to the Finkelstein theory,[25] both with (*1*) and without (*2*) allowance for weak localization effects.

$$\delta\sigma_{wl}(0,T)=\frac{e^2}{2\pi^2\hbar}p\ln\frac{T}{T_0}, \tag{5}$$

and the dependence on magnetic field for $B\ll B_{\rm tr}$, $B_\rho\ll B_{\rm tr}$ ($B_{\rm tr}=\hbar c/4eD\tau$; $B_\varphi=\hbar c/4eD\tau_\varphi$, $D$ is the diffusion coefficient, $\tau_\varphi$ is the dephasing time, which depends on the temperature as $T^{-p}$, where $p$ is an exponent determined by the scattering mechanism, dimensionality of the sample, etc.) is determined by the expression[28]

$$\delta\sigma_{wl}(B,T)=\frac{e^2}{2\pi^2\hbar}\left[\psi\left(\frac{1}{2}+\frac{B_\varphi}{B}\right)-\ln\frac{B_\varphi}{B}\right]. \tag{6}$$

Formula (6) describes the negative magnetoresistance due to the suppression of interference effects by the magnetic field. We emphasize that $\delta\sigma_{wl}$ depends only on the ratio $B/B_\varphi$, and for $p=1$ (the Nyquist mechanism) it is a function of the ratio $B/T$.

By comparing the dependence $\rho(T)$ in the region of "metallic" conductivity at $T<1$ K (see Fig. 1b) with expressions (2) and (5) for $p=1$ we see that such behavior is possible only when the predominant role is played by the antilocalization contribution of the triplet channel. A fitting gives $\lambda=0.68$, which corresponds to $\gamma_2=2.15$ (in the notation of Ref. 6, $F_0^\sigma=-\gamma_2/(1+\gamma_2)=-0.68$).

The magnetic field dependence (see Fig. 2b) can be described only by the joint influence of two effects: positive magnetoresistance due to the Zeeman splitting (3), and negative magnetoresistance due to dephasing (6), with a slight

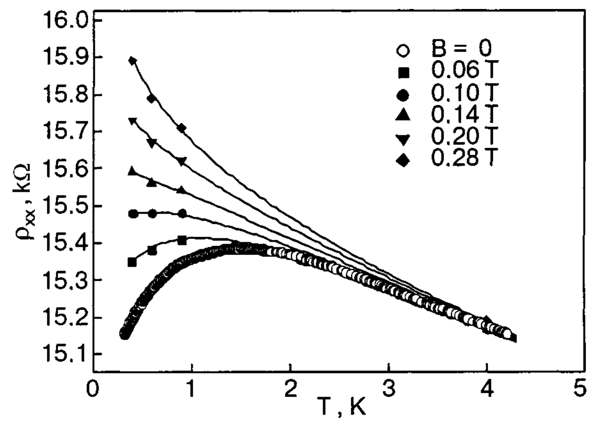


FIG. 4. Temperature dependence of the magnetoresistivity in fixed magnetic fields.

predominance of the first effect. For example, let us give the expression for $\delta\sigma=\delta\sigma_{ee}+\delta\sigma_{wl}$ at low fields $B\ll B_z=k_BT/g\mu_B$, $B\ll B_\varphi$:

$$\delta\sigma(B,T)=\frac{e^2}{2\pi^2\hbar}\left[-0.091\gamma_2(1+\gamma_2)+0.042\left(\frac{B_z}{B_\varphi}\right)^2\right]\times\left(\frac{B}{B_z}\right)^2, \tag{7}$$

where (for $p=1$) the ratio $B_z/B_\varphi$ is independent of temperature.

By fitting the dependence $\rho(B/T)$ using formulas (3) and (6) in the whole interval of magnetic fields one can determine separately the $g$ factor and $B_\varphi(\tau_\varphi)$. The value found for the $g$ factor, $g=14.2\pm1.4$, is somewhat lower than the theoretical value for $\varepsilon_F\to0$ ($=20.4$), in accordance with the high degree of nonparabolicity of the hole dispersion relation in the ground spatial subband. For the dephasing time a fit gives $k_BT\tau_\varphi/\hbar\approx1$, in good agreement with the theoretical estimate.

Simultaneously taking into account the disorder (localization effects) and the electron–electron interaction leads to renormalization of the parameter $\gamma_2$—to monotonic growth of $\gamma_2$ with decreasing temperature[25] (Fig. 3). As was shown in Ref. 24, such a renormalization is especially important in the region of the metal–insulator transition, which is determined by the condition $\varepsilon_F\tau/\hbar\approx1$. We assume that the nonmonotonic $\rho(T)$ dependence observed by us is due to just such a renormalization of the parameter $\gamma_2$ and, as a result, to a change in sign of the coefficient $(p+1-3\lambda)$ at $T\approx1.5$ K, although we have been unable to describe the effect quantitatively.

## CONCLUSIONS

Thus the observed nonmonotonic behavior of $\rho(T)$ and, specifically, the transition from insulating ($d\rho/dT<0$) to "metallic" ($d\rho/dT>0$) behavior with decreasing temperature is attributed by us to enhancement of the role of the triplet channel in the quantum correction to the conductivity due to the electron–electron interaction. The increase of the contribution of the triplet channel with decreasing temperature is apparently due to the renormalization of the electron–electron coupling parameter predicted in the Finkelstein theory,[25] which is especially substantial for 2D systems in the vicinity of the concentration-induced metal–insulator transition ($\varepsilon_F \tau/\hbar \approx 1$). The Zeeman splitting of the electron energy levels in a magnetic field leads to effective suppression of the triplet channel, thus restoring the insulating behavior of $\rho(T)$ down to the lowest temperatures (Fig. 4).

The authors thank V. I. Okulov and S. G. Novokshonova for a discussion of the results. This study was supported by the Russian Foundation for Basic Research, Grant No. 02-02-16401, 04-02-16614 and was carried out as part of the "Low-Dimensional Quantum Heterostructures" program of the Russian Academy of Sciences, the CRDF, and the Ministry of Education of the Russian Federation, Grant Y1-*P*-05-14 (Ek-005[X1]), and a grant for young scientists from the Ural Division of the Russian Academy of Sciences.

*E-mail: neverov@imp.uran.ru

1)The technological parameters of the sample were: number of periods (Ge+GeSi) $N=15$; quantum well (Ge layer) width $d_W=80$ Å, and barrier (GeSi layer) width $d_b=120$ Å.


[1] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[2] B. L. Altshuler and A. G. Aronov, in *Electron–Electron Interactions in Disordered Systems*, A. L. Efros and M. Pollak (eds.), Amsterdam (1985), p. 1.
[3] S. V. Kravchenko, G. V. Kravchenko, J. E. Furneaux, V. M. Pudalov, and M. D'Iorio, Phys. Rev. B **50**, 8039 (1994).
[4] B. L. Altshuler, D. L. Maslov, and V. M. Pudalov, Physica E **9(2)**, 209 (2001).
[5] E. Abrahams, S. V. Kravchenko, and M. R. Sarachik, Rev. Mod. Phys. **73**, 251 (2001); S. V. Kravchenko and M. P. Sarachik, cond-mat/0309140 (submitted to Rep. Progr. Phys.)
[6] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001); G. Zala, B. N. Narozhny, and I. L. Aleiner, cond-mat/0107333.
[7] I. V. Gornyi and A. D. Mirlin, Phys. Rev. Lett. **90**, 076801 (2003); I. V. Gornyi and A. D. Mirlin, cond-mat/306029.
[8] V. M. Pudalov, M. E. Gerhenson, N. Kojima, G. Brunthaler, A. Prinz, and G. Bauer, Phys. Rev. Lett. **91**, 126403 (2003); cond-mat/0401031.
[9] V. M. Pudalov, G. Brunthaler, and A. Prinz, and G. Bauer, JETP Lett. **65**, 932 (1997).
[10] D. Simonian, S. V. Kravchenko, M. P. Sarachik, and V. M. Pudalov, Phys. Rev. Lett. **79**, 2304 (1997).
[11] M. Y. Simmons, A. R. Hamilton, M. Perer, E. N. Linfield, R. D. Rose, D. A. Ritchie, A. K. Savchenko, and T. G. Griffiths, Phys. Rev. Lett. **80**, 1292 (1998).
[12] T. Okamoto, K. Hosoya, S. Kawaji, and A. Yagi, Phys. Rev. Lett. **82**, 3875 (1999).
[13] A. A. Shashkin, S. V. Kravchenko, V. T. Dolgorolov, and T. M. Klapwijk, Phys. Rev. Lett. **87**, 086801 (2001); cond-mat/0302004.
[14] S. A. Vitkalov, N. Zheng, K. M. Mertez, M. R. Sarachik, and T. M. Klapwijk, Phys. Rev. Lett. **85**, 2164 (2000).
[15] S. A. Vitkalov, K. James, V. N. Narozhny, and M. R. Sarachik, and T. M. Klapwijk, cond-mat/0204566.
[16] J. Yoon, C. C. Li, D. Shahar, D. S. Tsui, and M. Shayegan, cond-mat/9907128.
[17] N. Noh, M. R. Lilly, D. S. Tsui, J. A. Simmons, E. N. Hwang, S. Das Sarma, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **68**, 165308 (2003).
[18] V. T. Dolgopolov and A. Gold, JETP Lett. **71**, 27 (2000).
[19] O. Simonian, S. V. Kravchenko, M. R. Sarachik, and V. M. Pudalov, Phys. Rev. B **57**, R9420 (1998).
[20] J. C. Hensel and K. Suzuki, Phys. Rev. Lett. **22**, 838 (1969).
[21] Yu. G. Arapov, N. A. Gorodilov, O. A. Kuznetsov, V. N. Neverov, L. K. Orlov, R. A. Rubtsova, G. I. Kharus, A. L. Chernov, N. G. Shelushinina, and G. L. Shtrapenin, Fiz. Tekh. Poluprovodn. **27**, 1165 (1993) [Semiconductors **27**, 642 (1993)]; Yu. G. Arapov, O. A. Kuznetsov, V. N. Neverov, G. I. Kharus, N. G. Shelushinina, and M. V. Yakunin, Fiz. Tekh. Poluprovodn. **36**, 550 (2002) [Semiconductors **36**, 519 (2002)].
[22] A. V. Nenashev, A. V. Dvurechenskiĭ, and A. F. Zinov'eva, Zh. Éksp. Teor. Fiz. **123**, 362 (2003) [JETP **96**, 321 (2003)].
[23] P. T. Coleridge, A. S. Sachrajda, and R. Zawadzki, cond-mat/9912041; cond-mat/0011067; Phys. Rev. B **65**, 125328 (2001).
[24] A. Punnoose and A. M. Finkelstein, Phys. Rev. Lett. **88**, 016802 (2002).
[25] A. M. Finkelstein, Z. Phys. B **56**, 189 (1984).
[26] C. Castellani, C. Di Castro, and P. A. Lee, Phys. Rev. B **57**, R9381 (1998).
[27] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **65**, 020201 (2001).
[28] S. Hikami, A. I. Larkin, and I. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).


Translated by Steve Torstveit