Magnetotransport probing of the quality of the heterointerfaces and degree of symmetry of the potential profile of quantum wells in the valence band of the $Ge_{1-x}Si_x/Ge/Ge_{1-x}Si_x$ heterosystem

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(Submitted June 24, 2004)
Fiz. Nizk. Temp. **30**, 1139–1145 (November 2004)

It is shown that in a periodic system of p-type $Ge_{1-x}Si_x/Ge/Ge_{1-x}Si_x$ quantum wells having a Ge layer more than ~30 nm wide, the hole gas in each Ge layer is separated into two two-dimensional sublayers concentrated near opposite boundaries of the layer. This follows from the vanishing of the quantum Hall effect plateau and of the corresponding minimum of the longitudinal magnetoresistance for a filling factor $\nu=1$. Here positive magnetoresistance is observed, which is attributed to the presence of two types of holes with different mobilities. A quantitative analysis shows that these are mainly heavy holes having different mobilities in the sublayers that form. The difference of the mobilities indicates that the opposite heterointerfaces of the Ge layers are of different quality. It follows from an analysis of the shape of the quantum Hall effect plateau for $\nu=2$ that the densities of holes in the sublayers formed are close and, consequently, that the profile of the potential wells is close to symmetric. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1819860]

INTRODUCTION

Heterosystems consisting of layers of Si, Ge, and their solid solutions are of interest in connection with the significant improvements that have been achieved in the parameters of devices based on them (as compared with those made from the analogous bulk materials) for today's applications. Because of this, and also the relatively low cost of these materials, they are promising objects that can compete with other heterosystems.[1,2] For achieving high mobility of the carriers in the layer (which is essential for increasing the working frequency) an important role is played by the quality of the heterointerfaces: its geometric irregularities should be minimized, its sharpness should be maximized, and the fraction of impurities located on and near the interface should be minimized. This last factor is important, since high mobilities are achieved only through selective doping of the barriers, so that the ionized impurities are partitioned from the free carriers in the potential well by an undoped part of the barrier—a spacer. An impurity can diffuse from the nominal doping region and approach the heterointerface and even reach it. An increase in mobility can also be brought about by increasing the width of the well, since the average position of the carriers is then farther away from the heterointerface and from the doped region of the barriers. However, the latter process is prevented by the bending of the bottom of the well that occurs during growth of its width. For all of these reasons it is important to find methods for quality control of heterointerfaces and for monitoring the profile of the potential well.

In the heterosystems under study the potential well is predominantly formed in the valence band of the layer with the larger Ge content.[2] Therefore here the holes mainly form a size-quantized gas.

In studying the quantum Hall effect (QHE) we have found that in a multilayer system of p-type $Ge/Ge_{1-x}Si_x$ at a Ge layer thickness greater than ~30 nm the hole gas in each Ge layer separates into two two-dimensional sublayers.[3] On the one hand, this establishes a limit on the existence of a unified quasi-two-dimensional hole gas in the Ge layer (i.e., on the maximum distance of the holes from the heterointerface). On the other hand, since the holes are localized near one of the heterointerfaces in the two-dimensional sublayers that form, and the characteristics of that particular heterointerface influence the mobility of the holes near it, it becomes possible to carry out a comparative analysis of the opposite boundaries of the Ge layers.

SAMPLES AND EXPERIMENT

We shall present the results of measurements of the longitudinal $\rho_{xx}(B)$ and Hall $\rho_{xy}(B)$ magnetoresistivities (MRs) of multilayer $Ge/Ge_{1-x}Si_x$ samples with $x\approx 0.1$, grown on a substrate with the (111) orientation. The central part of the

TABLE I. Parameters of the $Ge/Ge_{1-x}Si_x$ multilayer samples.

Sample	N	d_w, nm	p_s, 10^{15} m^{-3}	μ, m^2/(V·5)
1006	90	12.5	4.9	1.4
1124b3	27	22	2.8	1.0
475a2	37	38	5	1.3
476a4	37	38	5	0.85
476b4	37	38	5.8	0.98

$Ge_{1-x}Si_x$ barriers is doped with boron. The samples differ in the width d_w of the Ge layers, the density p_s of the hole gas in them, and also the hole mobility μ and the number of $Ge/Ge_{1-x}Si_x$ periods N (for the parameters of the samples, see Table I).

The hole mobilities are rather high for observation of a clear picture of the QHE. Figure 1 shows a typical QHE pattern for Ge layers ~20 nm thick. (The resistivity in each of the figures is given per conducting Ge layer.) We call attention to the fact that the features of the QHE, i.e., the "shelves" on $\rho_{xy}(B)=h/e^2 i$ and the corresponding minima of $\rho_{xx}(B)$, are predominant at even values of i, but features are also observed for $i=1$. Negative magnetoresistance appears in the region of low magnetic fields (due to weak localization effects).

In the samples with Ge layer widths greater than ~30 nm the structure of the experimental curves changes substantially: the QHE feature for $i=1$ vanishes (Fig. 2), and a positive magnetoresistance appears which is expressed particularly clearly at low magnetic fields (Fig. 3).

SEPARATION OF THE HOLE GAS INTO TWO TWO-DIMENSIONAL SUBLAYERS

The vanishing of the QHE features for $i=1$ in samples with wide Ge layers indicates that the hole gas in the layers separates into two two-dimensional sublayers concentrated near opposite heterointerfaces.[4] The appearance of such an effect can be expected starting with a certain width of the potential well under the condition that its profile is symmetric (or at least that the deviations from symmetry are small). For the investigated series of samples with different widths of the Ge layer, the QHE feature for $i=1$ vanishes starting at

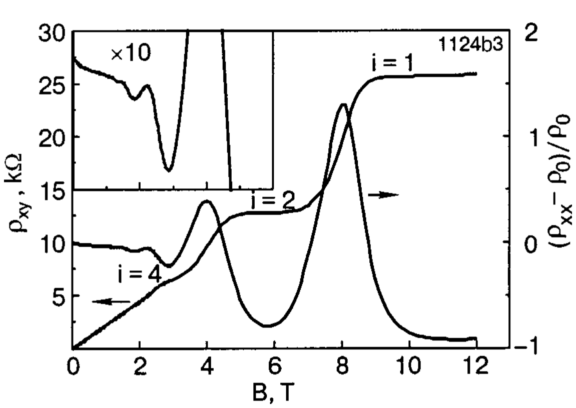

FIG. 1. Quantum Hall effect in sample 1124b3 ($d_w=22$ nm).

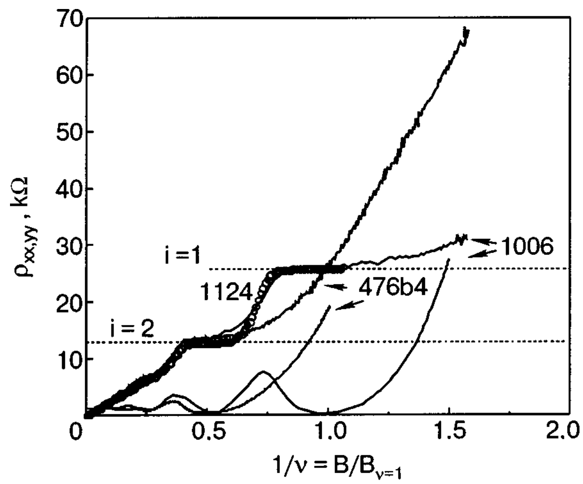

FIG. 2. Quantum Hall effect in Ge layers of different widths. The QHE features with $i=1$ vanish for a width of the Ge layer greater than ~30 nm.

$d_w \approx 30$ nm. The vanishing of this feature means that the profile of the potential well is symmetric, since otherwise in a wide asymmetric well the free carriers would simply leak into one of the triangular wells forming near the heterointerface. Such a situation is observed for one-sided doping of a potential well and for isolated wells found near the surface because of the influence of the charged surface states (see, e.g., Ref. 5). The symmetric nature of the profile of the potential wells in the samples that we investigated is probably promoted by their multilayeredness. The structure of the symmetric potential well for sample 475a2, calculated from the joint solution of the Schrödinger equation and Poisson's equation, is presented in Fig. 4. The Fermi level is found near the top of the curved bottom of this well, as is typical for the process of separation of a hole gas. However, the main sign of separation into two independent sublayers is the practically complete coalescence of the lower levels HH1 and HH2. The tunneling gap is practically absent (it amounts to ~1 μV) because of the large width of the barrier formed and the large mass of the heavy holes.

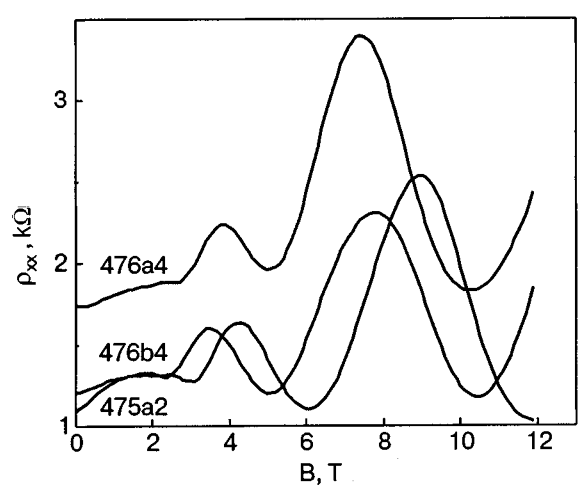

FIG. 3. Positive magnetoresistance in samples with a width of the Ge layer greater than 30 nm.

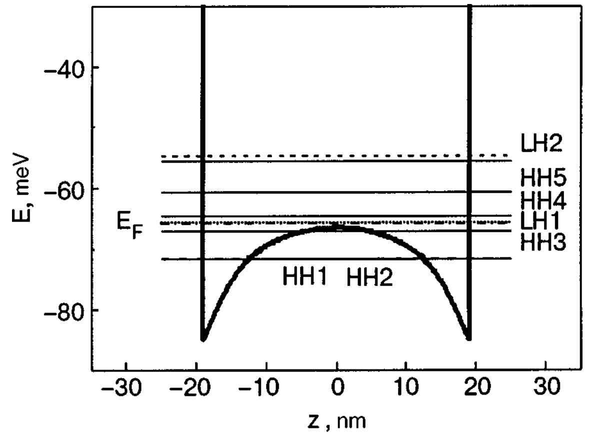

FIG. 4. Calculated potential profile, the energy levels, and the Fermi level of sample 475a2.

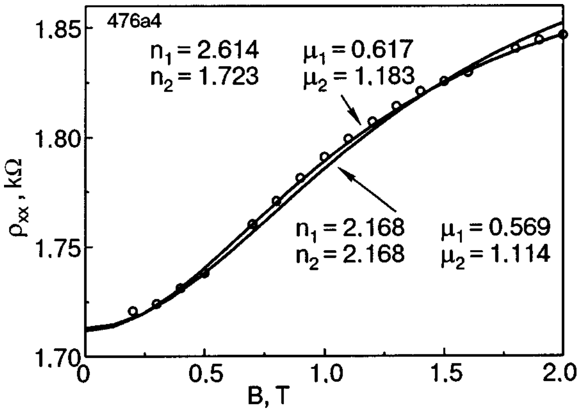

FIG. 5. Magnetoresistance of sample 476a4. Experiment (○); the solid curves were calculated according to formula (1) for two sets of parameters.

POSITIVE MAGNETORESISTANCE: A SIGN THAT THE OPPOSITE HETEROINTERFACES ARE DIFFERENT

The positive magnetoresistance in a perpendicular magnetic field is quite easily described by the participation of two types of carriers, with different mobilities, in the magnetotransport. The fact that positive magnetoresistance arises in samples with wide Ge layers in the same cases when the QHE feature with $i=1$ vanishes, i.e., when the hole gas separates into two two-dimensional sublayers, can be explained by the conjecture that holes with different mobility are found in the different sublayers. Thus the difference of their mobilities indicates that the opposite boundaries of the layer are of different quality. This difference is unsurprising, since these boundaries are formed under different conditions: the heterointerface of the Ge layer on the side farther from the substrate (the normal boundary) grows on a layer of pure elemental Ge, whereas the boundary nearer the substrate (the inverted boundary) grows on a layer of solid solution $Ge_{1-x}Si_x$ and, most importantly, that layer is doped. And although there is nominally an undoped spacer layer between the impurity layer and the heterointerface, in actuality an impurity can "float up" during growth, approaching and possibly even reaching the boundary with the Ge layer. It is therefore to be expected that the hole mobility is lower near the inverted heterointerface than near the normal boundary.[6] In relatively narrow layers every hole is sensitive to both boundaries, and therefore the holes all have the same mobility. Upon separation into hole sublayers in the case of a wide Ge layer the holes in each of the sublayers formed can have different mobilities.

The longitudinal and Hall magnetoresistivities in the presence of two types of carriers with different mobilities μ_j are described in the framework of the extremely simple Drude–Lorentz model by the formulas[7]

$$\rho_{xx}=(D_1+D_2)/[(D_1+D_2)^2+(A_1+A_2)^2],$$
$$\rho_{xy}=-(A_1+A_2)/[(D_1+D_2)^2+(A_1+A_2)^2]. \qquad (1)$$

Here $D_j=n_je\mu_j/(1+\mu_jB)$ is the diagonal term of the conductivity matrix for layer j, $A_j=\mu_jBD_j$ is the corresponding off-diagonal term, and n_j is the density of the 2D hole gas in sublayer j.

The curves of ρ calculated according to formula (1) for sample 476a4 are shown in Fig. 5. It is seen that both for equal densities of the hole gas in the layers, $n_1=n_2$, and also (with a somewhat better agreement with experiment) for $n_1 \neq n_2$ the mobilities in the sublayers must differ by roughly a factor of two.

It also follows from the results of the fitting that this method is rather insensitive to the difference in the densities of the hole gas in the sublayers. In the case of a large number of oscillations of $\rho_{xx}(B)$, which occurs, e.g., in the conventional heterosystem of n-type GaAs/AlGaAs,[8,9] the difference between n_1 and n_2 can be determined from two different series of oscillations. In a hole gas, however, this cannot be done.

ESTIMATE OF THE RATIO OF THE DENSITIES OF THE HOLE GAS IN THE SUBLAYERS FROM THE STRUCTURE OF THE QHE

Let us first answer the question of how the QHE feature can be realized for the Ge layer as a whole at a definite value of i $(=2)$ in the face of such a large difference in the mobilities in the sublayers. Even at equal hole densities in the two sublayers the difference of their resistivities will be proportional to the difference of the hole mobilities, and when the sublayers are connected in parallel into a common circuit the currents flowing along the sublayers will differ by the same factor. If two given sublayers are considered to be isolated from each other, then in each of them the first plateau of $\rho_{xy}(B)$ from the high-field end will correspond to $i=1$, i.e., will have a height $\rho_{xy}=25.813$ kΩ. However, the voltages $U^l_{xy}=\rho_{xy}I^l$ corresponding to them will be different on account of the difference of the currents I^l (the index l numbers the layer). Then if one considers the classical situation for an initially uniform distribution of the current streamlines over the width of the sample, then when the corresponding potential contacts of the sublayers are connected, because of the potential difference across these contacts circulating currents should flow between them, distorting the pattern of streamlines in each sublayer. After these contacts are connected the potentials across them change, and a calculation of the value of the potential difference formed across such

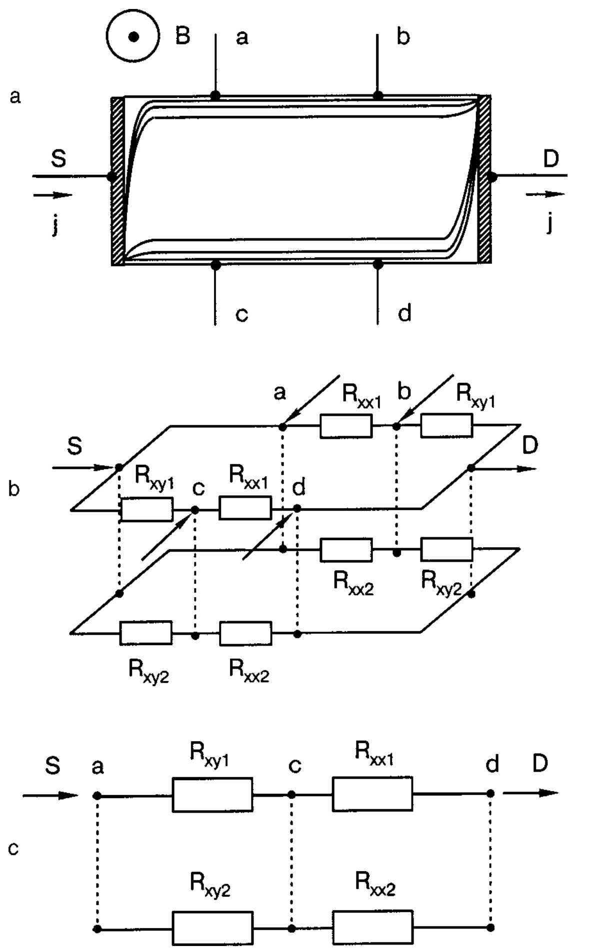

FIG. 6. Equivalent circuit of a system of two layers in the QHE regime. The current streamlines in an isolated 2D layer (a); the distribution of the Hall and longitudinal resistances in the layer (b); the layer resistances connected in parallel (c).

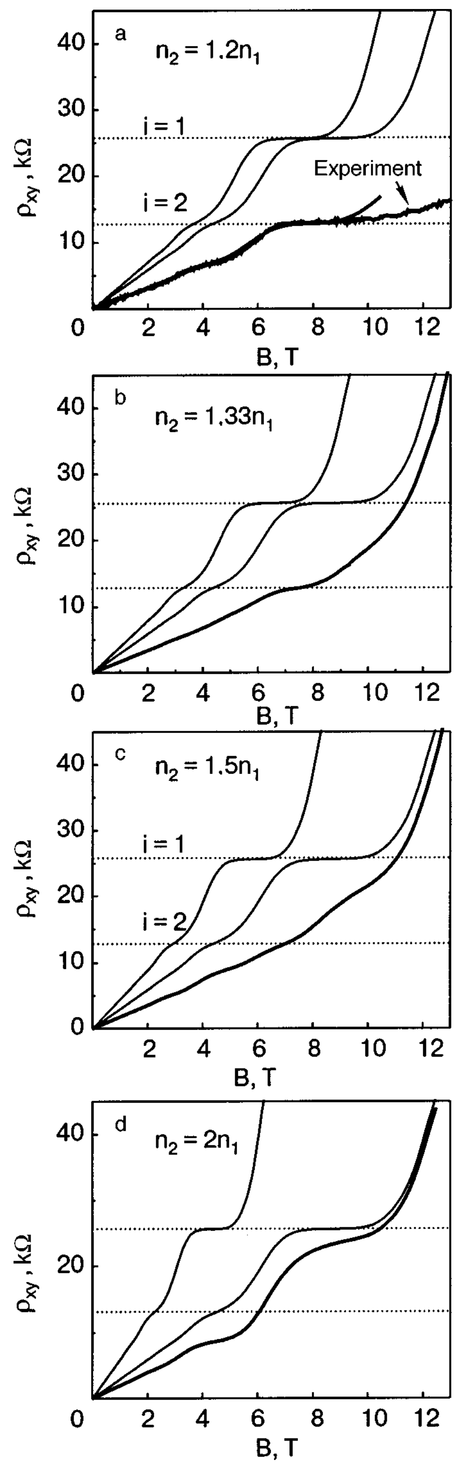

FIG. 7. Transformation of the resultant Hall resistance of two parallel layers for increasing differences of the hole concentrations in them. The top two curves are for the individual layers; the lower curve is the result of their connection in parallel. a) $n_2=1.2n_1$; the resultant model curve is compared with the experimental curve for sample 475b4.

joined Hall contacts in a circuit with distributed parameters and currents circulating in the transverse direction is, generally speaking, not a simple problem.

In actuality, in the QHE regime the classical picture of the uniform distribution of current over the cross section does not apply. At zero temperature ideal 2D layers (i.e., in the complete absence of any parallel 3D conduction) in the interval of magnetic fields corresponding to the QHE the potential along the whole perimeter of the sample can take on only two values, so that the difference of these potentials is equal to h/ie^2 (Ref. 10). In this case discontinuity of the equipotential lines occurs only at two points—at the inlet and outlet of the current.[11] This means that even when the end of the sample is completely coated with a conducting material, the current will flow into the body of the sample only at one point—at the edge of the contact of the conducting coating with the sample, and it will flow out only at a single diagonally opposite point (see Fig. 6a, which shows the current streamlines in a rectangular bar in the QHE regime).

In the equivalent circuit of the 2D layer the Hall resistance will be represented by two resistors R_{xyl} corresponding to the aforementioned two discontinuities of the equipotentials (Fig. 6b). Indeed, as is seen in Fig. 6b, the voltage drop across the left resistor R_{xy} in a layer taken separately is equal to the voltage between contacts a–c, and the voltage drop across the right resistor R_{xy} is equal to the voltage between contacts b–d. Thus the Hall potential difference in the QHE regime occurs in a vanishingly small region on the perimeter of the sample. In this scheme all the deviations from the ideal QHE regime, responsible for the nonzero value of the longitudinal resistance in the region of the QHE plateau, accumulate as the current flows along the perimeter from the inlet point to the outlet point. This part is represented on the equivalent circuit by the resistors R_{xx}. It is these resistors that reflect the finite value of the mobility in the QHE plateau region in a real sample. The given equivalent circuit shows that in the QHE regime one can get away from a problem with distributed parameters. As a result, for layers connected in parallel the Hall resistances R_{xyl} of the different layers turn out to be connected in parallel with each other, just as the longitudinal resistances R_{xxl} are (Fig. 6c). Importantly, here the resultant resistances R_{xy} and R_{xx} turn out to be completely decoupled: they are mutually independent and uniquely determined by the corresponding single-layer components.

Thus for a parallel connection of the 2D layers in the QHE regime the differences in the mobility of the carriers in the individual layers has no effect on the resultant value of the Hall resistance of the system. Consequently, for finding the resultant Hall resistance of the sublayers one can treat them as equivalent irrespective of the differences in mobility. Therefore, in the case of a parallel connection of two identical 2D layers ($n_1 = n_2$) in the magnetic field interval in which each isolated layer has a QHE plateau of height $\rho_{xy} = h/e^2 = 25.814$ kΩ, a two-layer system will have a plateau with $\rho_{xy} = h/2e^2 = 12.907$ kΩ.

If the densities of the two-dimensional carrier gas in the layers are different ($n_1 \neq n_2$), then a two-layer system will exhibit a shortened plateau with $\rho_{xy} = h/2e^2 = 12.907$ kΩ in the field interval in which for the isolated layers the shelves overlap. In the two-layer system the shelf with $\rho_{xy} = h/e^2 = 25.814$ kΩ will be absent (see Fig. 7, where we present the results of a mathematical modeling of this situation).

With increasing difference of the densities of the 2D gas in the sublayers the plateau that initially was clearly discernable around $i = 2$ for the system (Fig. 7a) is first smeared out (Fig. 7b,c) and then reforms, but now around $i = 1$ (Fig. 7d). Our experimental results for sample 475b4 (Fig. 7a) coincide with the resulting model curve for $n_2 = 1.2n_1$ over a wide range of magnetic inductions, so the densities of the 2D gas in the sublayers are not very different. In any case the difference is less than 20%, confirming the conclusion reached above that the potential well has a symmetric profile.

It should be mentioned again that the modeling done is valid only in the fields regions where the constituent sublayers have a QHE plateau on their $\rho_{xy}(B)$ curves. At low fields it is more correct to describe the curves on the basis of formula (1).

CONCLUSION

It has been established from QHE studies that the spontaneous formation of the potential profile of a double quantum well occurs in p-type doped Ge/Ge$_{1-x}$Si$_x$ heterosystems having widths of over ~30 nm. On the one hand, this opens up some possibilities for seeking and investigating the features inherent to intercoupled two-dimensional layers (i.e., for studying interlayer correlated states). On the other hand, galvanomagnetic effects in this regime can be used for a comparative analysis of the quality of the heterointerfaces on opposite sides of the conducting layer and for assessing the degree of symmetry of its potential profile.

It has been found that the samples had two types of holes with mobilities differing by roughly a factor of two. These cannot be heavy and light holes, since their densities are close, and this effect is not observed in narrow layers of the same heterosystem. Consequently, these are holes localized near the opposite heterointerfaces, and the difference of their mobilities means that the quality of these heterointerfaces is substantially different. This illustrates the possibility of performing quality control of the opposite boundaries of the layer from magnetoresistance studies.

We have shown that the potential profile of the quantum wells in a multilayer system can be symmetric from the start, unlike the known situation for isolated conducting layers, where an external electric field must be applied with the aid of a gate in order to compensate the asymmetry induced by the charges localized on the surface.[12]

This study was supported by the Russian Foundation for Basic Research, Projects 02-02-16401, 04-02-16614 and the ‘‘Physics of Solid State Nanostructures’’ program of the Russian Academy of Sciences.

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Translated by Steve Torstveit