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# Scaling behavior of metal–insulator transitions in a Si/SiGe two dimensional hole gas

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## Abstract

We report the temperature dependence of magnetotransport measurements in a Si/SiGe two-dimensional hole gas (2DHG) and we analyze the curves in terms of scaling. A reentrant insulating transition is observed at filling factor  $\nu=1.5$ , followed by a second high field insulating phase at  $\nu < 1$ . A scaling behavior in temperature of the width of the longitudinal conductivity, its second derivative and the slope of the Hall conductivity has been observed, for both the transitions to the insulating state. © 2002 Elsevier Science B.V. All rights reserved.

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In a magnetic field, the conductance of a two-dimensional electron system shows vanishing conductivity and quantized Hall plateau, when an integer number of Landau levels are filled. In samples characterized by a short-range scattering potential, like  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{InP}$ , the transitions between two adjacent plateaus (PP transitions) and between the lowest Landau level and the insulating state in the extreme quantum limit (PI transition) can be all described as quantum critical phenomena [1,2]. In particular, in  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{InP}$ , the slope of the Hall conductivity and the width of the longitudinal conductivity have been shown to have all the same temperature

dependence  $\propto T^{-\kappa}$ , while the value of the second derivative of the longitudinal conductivity shows a power law dependence on  $T$  with a critical exponent of  $2\kappa$  [1–3]. For this sample, it was found that the critical exponent is  $\kappa = 0.42 \pm 0.04$  and is independent on the Landau level index. Despite the theoretical predictions of the universality of the scaling behavior, such a behavior has been observed only in a few experiments.

In this paper, we report magnetotransport measurements of a low mobility strained p-type  $\text{Si}_{0.88}\text{Ge}_{0.12}/\text{Si}$  heterostructure [4],<sup>1</sup> which has the same short-range scattering potential as  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{InP}$ , but a completely different energy level structure. This sample shows not only a Hall-insulator transition in the

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<sup>1</sup> The present sample is from the growth CVD191 in this paper.

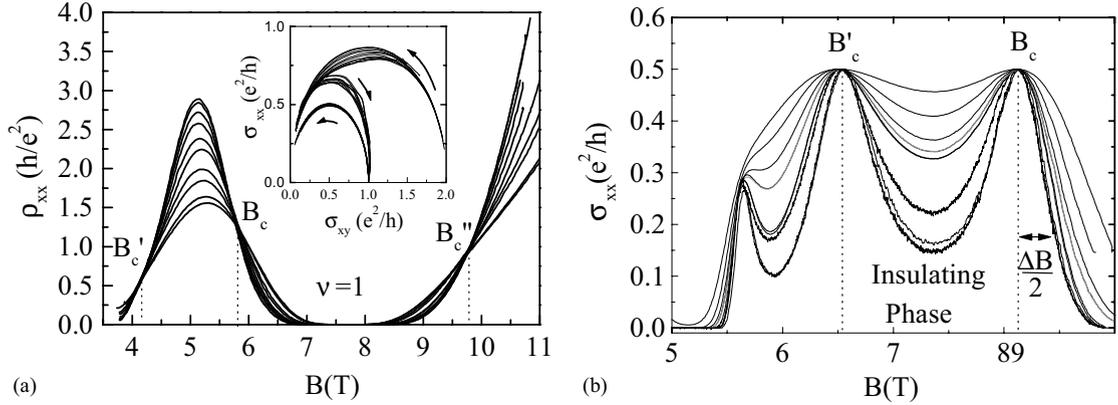


Fig. 1. (a) Longitudinal magnetoresistivity at different temperatures ( $0.3 \text{ K} < T < 1.2 \text{ K}$ ) for a carrier density  $p = 1.76 \times 10^{15} \text{ m}^{-2}$ . The inset shows  $\sigma_{xx}$  as a function of  $\sigma_{xy}$  and demonstrates how the conductivity enters and leaves the insulating state at  $\nu = 1.5$  for different temperatures. (b) The longitudinal conductivity  $\sigma_{xx}$  for the insulator transition at filling  $\nu = 1.5$  in the temperature range 70–850 mK ( $p = 2.42 \times 10^{15} \text{ m}^{-2}$ ).

quantum limit ( $\nu < 1$ ) but also an insulating phase around  $\nu = 1.5$ , not seen in  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{InP}$ . This transition has been observed also in other samples [5,6] and it is considered to be associated to the spin degeneracy of two Landau levels [7–9].

In Fig. 1a, the longitudinal magnetoresistivity is plotted at different temperatures. There are three values of the field ( $B'_c$ ,  $B_c$ , and  $B''_c$ ) where the resistivity does not depend on temperature: these three fields, called critical fields, separate the Hall states ( $B < B'_c, B_c < B < B''_c$ ) from the insulating phases ( $B'_c < B < B_c, B > B''_c$ ). In order to study the temperature scaling of the two insulating transitions, we converted the  $\rho_{xx}$  and  $\rho_{xy}$  into the longitudinal ( $\sigma_{xx}$ ) and the Hall ( $\sigma_{xy}$ ) conductivity, through the inversion of the transport matrix. Fig. 1b shows  $\sigma_{xx}$  calculated for  $p = 2.42 \times 10^{15} \text{ m}^{-2}$  between filling factor 2 and 1:  $\sigma_{xx}$  vanishes in both the insulating and in the Hall states and reaches its maximum value at the critical field  $B_c$  and  $B'_c$ . The analysis of the conductivity, which is always finite in the metal–insulator transition, proves to be more accurate than the analysis of the resistance, which diverges in the insulating state. This might explain the different values of the exponents reported in earlier scaling analysis [10,11] and obtained by overlapping the resistance curves. In this paper, we analyze the conductivity measurements over a wide temperature range for both the longi-

nal and the Hall conductivity in terms of scaling and we obtain the same critical exponent for the two Hall insulating transitions.

In our measurements, different cooling procedures allow experiments at different carrier density ( $p = 1.76 \times 10^{15}$  and  $2.42 \times 10^{15} \text{ m}^{-2}$ ) with the same sample. Fig. 1a shows the magnetoresistance curves for  $p = 1.76 \times 10^{15} \text{ m}^{-2}$  in magnetic field up to 12 T and in the temperature range 300 mK–1.2 K. The flow diagram [6] in the inset of Fig. 1a shows the transition from  $\nu = 2$  to insulating state where  $\sigma_{xx} \rightarrow 0$  (as the field increases), the insulating state goes to a Hall state at  $\nu = 1$  at higher fields, and, in the end, there is the insulating transition in the quantum limit ( $\nu < 1$ ). The scaling theory predicts the width of the conductivity  $\Delta B$  (defined as in Fig. 1b) to vanish with temperature with a power law  $T^\kappa$ . Moreover, the same theory predicts that the slope of the Hall conductivity  $(\partial\sigma_{xy}/\partial B)_{B=B_c}$  diverges at  $T = 0$  with the same critical exponent. In our measurements, the analysis of the longitudinal conductivity  $\sigma_{xx}$  allows to calculate the high field half-width of the conductivity ( $\Delta B/2$ ) for the IP transition centered around the critical field  $B_c$ , which is not influenced by the proximity of the critical field  $B'_c$  of the PI transition (Fig. 1b). The temperature dependence of the width of the conductivity  $\Delta B$  is the same for IP and PI transitions and the critical exponent  $\kappa$  in both the cases  $\kappa = 0.45 \pm 0.05$

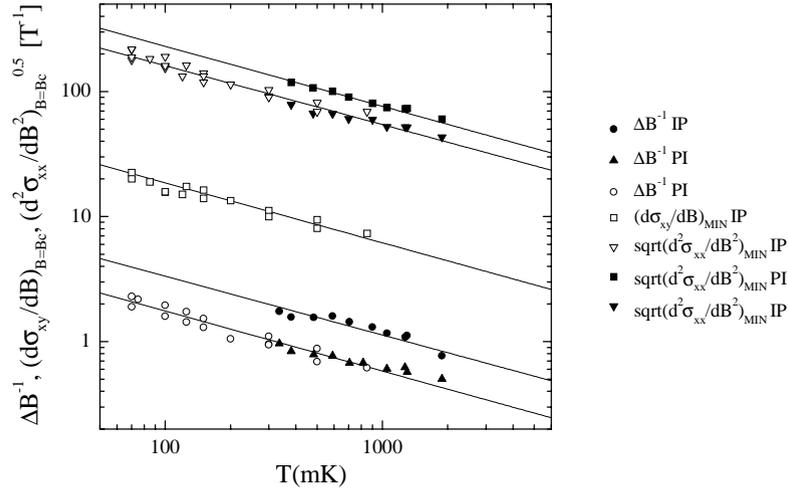


Fig. 2. The temperature scaling of  $1/\Delta B$ ,  $(\partial\sigma_{xy}/\partial B)_{B=B_c}$ , and square root of  $(\partial^2\sigma_{xx}/\partial B^2)_{B=B_c}$  in a  $\text{Si}_{0.88}\text{Ge}_{0.12}/\text{Si}$  heterostructure for both the high field plateau–insulator (PI) at  $\nu < 1$  and the reentrant insulator–plateau (IP) transitions at  $\nu = 1.5$ . The solid symbols are data taken for a hole density  $p = 1.76 \times 10^{15} \text{ m}^{-2}$ , while the open symbols are measurements at  $p = 2.42 \times 10^{15} \text{ m}^{-2}$ . The slope of the straight line gives critical exponent:  $\kappa = 0.45 \pm 0.05$ .

(Fig. 2). The value of the critical exponent  $\kappa$  in the IP transition is  $\kappa = 0.45 \pm 0.05$  for both the densities. The analysis of the slope of the Hall conductivity  $(\partial\sigma_{xy}/\partial B)_{B=B_c}$  for  $p = 2.42 \times 10^{15} \text{ m}^{-2}$  (Fig. 2) shows that  $(\partial\sigma_{xy}/\partial B)_{B=B_c} \propto T^{-\kappa}$ , with  $\kappa = 0.45 \pm 0.05$ .

According with the scaling theory, if the width of the conductivity  $\Delta B$  exhibits a scaling behavior in temperature with an exponent  $\kappa$ , the second derivative calculated at the critical field should diverge approaching  $T = 0$  with the power law  $T^{-2\kappa}$ . In our measurements, the calculation of the second derivative of the longitudinal conductivity  $(\partial^2\sigma_{xx}/\partial B^2)_{B=B_c} \propto T^{-2\kappa}$  for both the PI and IP transitions confirms the value of the critical exponent already found for  $\Delta B$ :  $\kappa = 0.45 \pm 0.05$ . Fig. 2 summarizes all the results for the exponents of different transitions and densities, obtained by the analysis of experimental curves at different temperatures. It can be seen that the transition to the insulating state and the reentrant insulating phase transition all exhibit the same scaling behavior, with the same critical exponent:  $\kappa = 0.45 \pm 0.05$ . The critical exponents of the  $\sigma_{xx}$  width, the slope of  $\sigma_{xy}$  and the second derivative of  $\sigma_{xx}$  are the same and, within the experimental error, in agreement with the critical exponents previously observed in  $\text{InGaAs}/\text{InP}$  heterostructures [1–3].

In summary, through an accurate scaling analysis over a wide temperature region, we observed scaling properties for both the reentrant Hall insulator transition at  $\nu = 1.5$  and the high field insulating phase transition, at two different densities. Our analysis is based not only on the value of  $\sigma_{xx}$  width but also on the first and second derivatives of the conductivity. Since the two transitions show the same scaling behavior, with the same critical exponent, they belong to the same universality class.

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