

## Probing the Plateau-Insulator Quantum Phase Transition in the Quantum Hall Regime

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(Received 2 December 1998)*

We report quantum Hall experiments on the plateau-insulator transition in a low mobility  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{InP}$  heterostructure. The data for the longitudinal resistance  $\rho_{xx}$  follow an exponential law and we extract a critical exponent  $\kappa = 0.55 \pm 0.05$  which is slightly different from the established value  $\kappa = 0.42 \pm 0.04$  for the plateau transitions. Upon correction for inhomogeneity effects, which cause the critical conductance  $\sigma_{xx}^*$  to depend marginally on temperature, our data indicate that the plateau-plateau and plateau-insulator transitions are in the same universality class.

PACS numbers: 73.40.Hm, 72.10.-d, 73.20.Dx

In the field of two dimensional electron gases the nature of the transitions between adjacent quantum Hall plateaus (*PP* transition) is an ardent topic of research. Experiments on low mobility  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{InP}$  heterostructures are a remarkable demonstration of a *quantum phase transition* indicating that the quantum Hall steps become infinitely sharp as the temperature ( $T$ ) approaches absolute zero [1,2]. The maximum slope in the Hall resistance ( $\rho_{xy}$ ) with varying magnetic field  $B$  was shown to diverge algebraically in  $T$ ,  $(d\rho_{xy}/dB)_{\text{max}} \propto T^{-\kappa}$  with the critical exponent  $\kappa = 0.42 \pm 0.04$ . On the other hand, the half-width ( $\Delta B$ ) of the longitudinal resistance ( $\rho_{xx}$ ) was shown to vanish like  $\Delta B \propto T^\kappa$  with the same exponent  $\kappa$ .

Because of the short range random alloy potential scattering, the low mobility  $\text{In}_x\text{Ga}_{1-x}\text{As}/\text{InP}$  structure has proven to be exceptionally important for studying scaling phenomena. This produces a wide range in  $T$  where the transport is dominated by Anderson (de)localization effects. This is in sharp contrast to high mobility  $\text{GaAs}/\text{Al}_x\text{Ga}_{1-x}\text{As}$  heterostructures where the long range potential fluctuations dramatically complicate the observability of the critical phenomenon, given the limitations of the experiment [3,4].

Nevertheless, the *PP* transitions in  $\text{GaAs}$  heterostructures have been studied extensively. It is therefore not surprising that the *universality* of the plateau transitions has not been established by these investigations. In these experiments, the same value of  $\kappa \approx 0.42$  was found but for a few samples only and for a small range in lowest  $T$  [4]. However, in most of the samples simple data fitting produced  $\kappa$ 's ranging from 0.2 up to 0.9. These results are

Landau level dependent and even for a given Landau level the  $\rho_{xx}$  and  $\rho_{xy}$  data give rise to different values for  $\kappa$  [5].

The focus in the last few years has been on transport in the lowest Landau level. For this purpose samples of lower density were used [6–10]. The bare resistance data look quite different from those of the other Landau levels since the transition is now between a quantum Hall (plateau) phase and an *insulator* (*PI* transition). There are striking similarities between the *PI* transition at high  $B$  and the superconductor-insulator transition [11] and the metal-insulator transition in two dimensions at  $B = 0$  [12].

One of the most important predictions of the renormalization theory is that the *PP* and *PI* quantum phase transitions are the same [2]. This stipulates that the same  $\kappa$  be observed as  $T$  approaches absolute zero and that the (electron-hole) symmetry in the  $\sigma_{xx}$ ,  $\sigma_{xy}$  conductance plane be retained. These aspects of Landau level systems serve as an important guide for selecting samples, particularly since the range of available  $T$  in transport measurements is rather limited and usually not sufficient to establish a critical point. However, in the experiments of Refs. [6–10] a comparison between the *PP* and *PI* transitions within the same sample was either not possible or not drawn. Moreover, these experiments provide inconclusive results because the samples used suffer from the aforementioned complications.

Recently, an interesting empirical result for the lowest Landau level  $\rho_{xx}$  has been reported [13]. For arbitrary samples at finite  $T$ , the  $\rho_{xx}$  data seems to depend linearly rather than algebraically on  $T$ , indicating that the problem is generically the same for all  $\text{GaAs}$  samples. Once

again, the experimental design has overlooked an essential requirement for studying scaling phenomena: the importance of short range random potential scattering—an essential (but certainly not sufficient [14]) prerequisite for sample choice.

In this paper we report the results of magnetotransport experiments performed on an  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{InP}$  heterostructure. Our main objective is to study critical aspects of the  $PI$  transition and to compare the results to the  $PP$  transitions measured on the same sample. We benefit from the fact that our sample has been studied before [15]. In particular, the exponent  $\kappa$  for the  $PP$  transitions was found to be 0.42 and 0.20 for spin polarized and spin degenerate Landau levels, respectively. The mobility of the sample is  $\mu = 16000 \text{ cm}^2/\text{Vs}$  measured at  $T = 4.2 \text{ K}$ . The electron density is  $2.2 \times 10^{11} \text{ cm}^{-2}$  which means that the  $PI$  transition occurs at  $B \approx 16 \text{ T}$ .

Our experiments were carried out in a Bitter magnet ( $B < 20 \text{ T}$ ) using a plastic dilution refrigerator (0.1–2 K) and a bath cryostat (1.5–4.2 K). The magnetotransport properties were measured using a standard ac technique with a frequency of 6 Hz and an excitation current of 5 nA. The main experimental results are presented in Fig. 1, where we have traced  $\rho_{xx}$  and  $\rho_{xy}$  as  $B$  sweeps through the  $PI$  transition. The different sets of data show a characteristic value  $B \approx 16 \text{ T}$  separating the insulating phase at high  $B$  and the quantum Hall phase at lower  $B$ . The  $\rho_{xy}$  data are shown in the inset of Fig. 1. This quantity, at low  $T$ , is clearly not quantized through the transition.

In order to recognize the relevant structure in the combined  $\rho_{xx}, \rho_{xy}$  data we have computed the conductance components  $\sigma_{xx}$  and  $\sigma_{xy}$  in the standard fashion. The results for different  $T$  are plotted in the  $\sigma_{xx}, \sigma_{xy}$  flow diagram (Fig. 2, upper inset). The symmetry about the

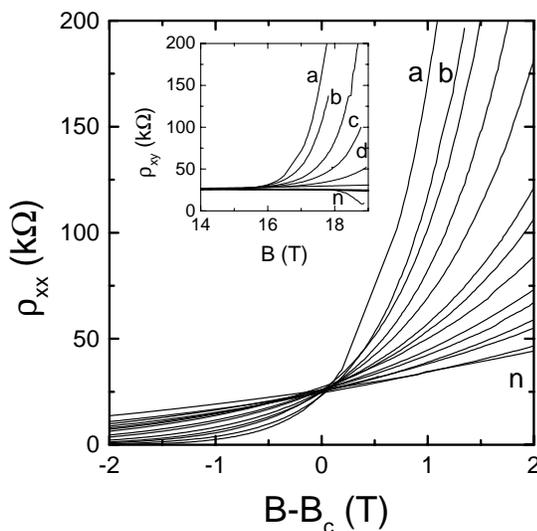


FIG. 1.  $\rho_{xx}$  and  $\rho_{xy}$  data with varying  $B$ . The curves are labeled  $a, b, \dots, n$  and the corresponding  $T$ 's are 0.13, 0.21, 0.26, 0.35, 0.47, 0.59, 0.83, 1.04, 1.4, 1.5, 1.9, 2.2, 3.1, and 4.2 K.  $B_c$  is the critical  $B$  (see text).

line  $\sigma_{xy} = 1/2$  is striking and reflects the high quality of the experimental data. The same symmetry was observed and discussed in the original work on the  $PP$  transitions [16]. Following the renormalization theory of the quantum Hall effect [2] we extract the critical value  $B_c$  from the maximum in  $\sigma_{xx}$  or from  $\sigma_{xy} = 1/2$ . The  $B_c$  ranges from 16.3 T at 0.13 K up to 16.9 T at 4.2 K, indicating a change in electron density of 3.5%. This is smaller than what is obtained from the low  $B$  Hall data, which gives a density of  $2.0 \times 10^{11} \text{ cm}^{-2}$  at 0.13 K as compared to  $2.2 \times 10^{11} \text{ cm}^{-2}$  at 4.2 K. The symmetry in the  $\sigma_{xx}, \sigma_{xy}$  diagram is a direct consequence of the following relations which hold at low but fixed  $T$ :

$$\begin{aligned}\sigma_{xx}(\Delta\nu) &= \sigma_{xx}(-\Delta\nu); \\ \sigma_{xy}(\Delta\nu) &= 1 - \sigma_{xy}(-\Delta\nu).\end{aligned}\quad (1)$$

Here  $\Delta\nu = \frac{1}{B} - \frac{1}{B_c} = \nu - \nu_c$ . We have explicitly verified the validity of Eq. (1). This result is important since it fundamentally reflects the electron-hole symmetry in the problem. Our data do not follow the statement of “duality” [17,18] which says that  $\rho_{xx}(\Delta\nu) = \rho_{xx}^{-1}(-\Delta\nu)$  and  $\rho_{xy}$  remains quantized through the  $PI$  transition. Instead, we observe that the  $\sigma_{xx}$  peak at  $\Delta\nu = 0$  develops a maximum around 1 K. Thus only for  $T < 1 \text{ K}$  the asymptotic scaling regime for the  $PI$  transition is attained [1]. For the  $PP$  transitions this range is larger ( $T < 3 \text{ K}$ ).

Since the  $\sigma_{xx}, \sigma_{xy}$  data show the characteristic Landau level independent behavior, we can now extract the

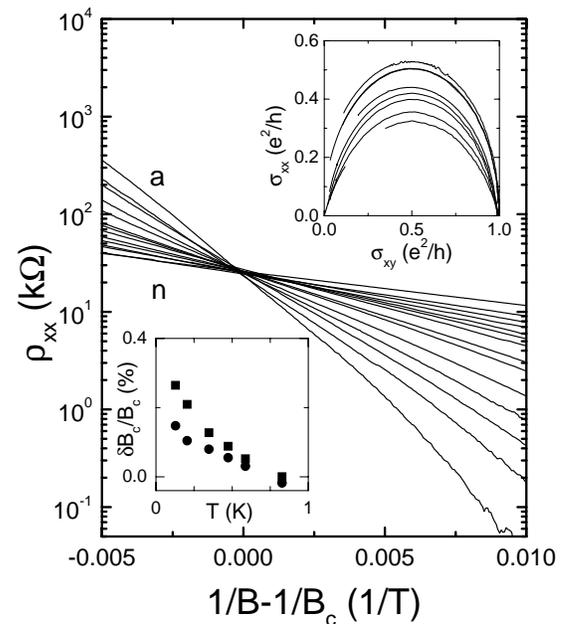


FIG. 2.  $\rho_{xx}$  data on a log scale with varying  $1/B$ . The labels and  $T$ 's are the same as in Fig. 1. Upper inset:  $\sigma_{xx}$  vs  $\sigma_{xy}$  at various  $T$  in the range 1 K–130 mK. Lower inset:  $\delta B_c/B_c$  vs  $T$ . The squares are the data derived from  $\rho_{xx}$ , Eqs. (2) and (4), and the circles are the data obtained from the different definitions of  $B_c$  (see text).

critical exponents in a similar fashion as done previously with the  $\rho_{xx}$  and  $\rho_{xy}$  data from the higher Landau levels. For example, from  $\sigma_{xx}$  with varying  $B$  we obtain the half-width  $\Delta B \propto T^\kappa$  with an exponent  $\kappa = 0.46 \pm 0.05$ . On the other hand, we find  $(\frac{d\sigma_{xy}}{dB})_{\min} \propto T^{-\kappa}$  with  $\kappa = 0.43 \pm 0.05$ . We attribute the small difference between the exponents to the uncertainties caused by mixing the  $\rho_{xx}, \rho_{xy}$  data in the computation of the conductances [16]. In Fig. 3 we plot the  $(\Delta B)^{-1}$  vs  $T$  for both the  $PI$  and the  $2 \rightarrow 1$  plateau transition. The latter was derived from the half-width in  $\rho_{xx}$ . For later purposes we have also plotted the low  $T$  data for  $(\frac{d\sigma_{xy}}{dB})_{\min}$  vs  $T$  in the lower inset of Fig. 4. The resulting exponents  $\kappa = 0.46 \pm 0.05$ ,  $0.42 \pm 0.05$ , and  $0.43 \pm 0.05$  are all the same, within the experimental error, indicating that the  $PP$  and the  $PI$  transitions are in the same universality class.

Next we compare our data to the exponential result [13]  $\rho_{xx}(\nu, T) \propto \exp[-\Delta\nu/\nu_0(T)]$ . In Fig. 2 we have traced  $\rho_{xx}$  on a log scale as a function of the difference  $\Delta\nu$ . This gives an adequate description of the data. The slope ( $\nu_0^{-1}$ ) of the straight lines at the transition regime can be accurately determined at each  $T$ . In Fig. 3 we plot  $\nu_0^{-1}$  vs  $T$  on a log-log scale. The data nicely follow the algebraic behavior  $\nu_0^{-1} \propto T^{-\kappa'}$  with  $\kappa' = 0.55 \pm 0.05$ . This value differs from the expected value  $\kappa = 0.42 \pm 0.05$  by more than the experimental error.

It is important to note that the data do not follow the linear behavior  $\nu_0 = \alpha T + \beta$  as proposed and advocated in Ref. [13]. Such linear dependence on  $T$  clearly does not describe the asymptotics of the quantum phase transition at  $T$  equal to zero. Instead, it is semiclassical in nature and typically observed at finite  $T$  and on samples with predominantly slowly varying potential fluctuations [3].

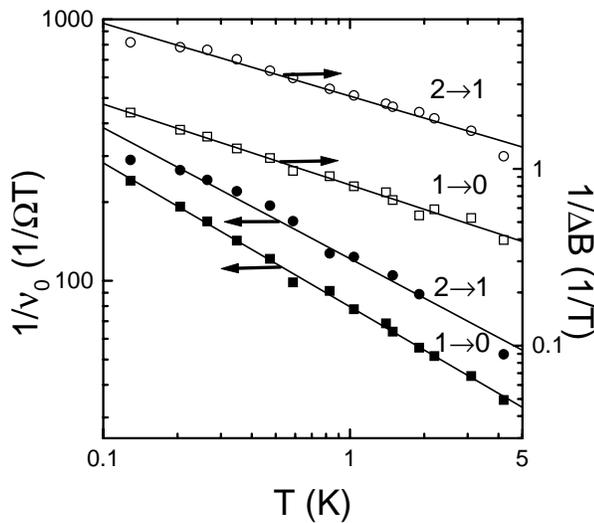


FIG. 3. Left axis:  $1/\nu_0$  vs  $T$  for the  $2 \rightarrow 1$  plateau transition (full circles,  $\kappa' = 0.51$ ) and the  $PI$  transition (full squares,  $\kappa' = 0.55$ ). Right axis:  $1/\Delta B$  vs  $T$  for the  $2 \rightarrow 1$  plateau transition (open circles,  $\kappa = 0.42$ ) and the  $PI$  transition (open squares,  $\kappa = 0.46$ ).

In order to show that the value  $\kappa' = 0.55 \pm 0.05$  is not a specific property of the  $PI$  transition, we have mapped the  $2 \rightarrow 1$  plateau transition onto the lowest Landau level following the steps  $\rho_{xx}, \rho_{xy} \rightarrow \sigma_{xx}, \sigma_{xy} \rightarrow \sigma_{xx}, \sigma_{xy} - 1 \rightarrow \rho_{xx}, \rho_{xy}$ . The  $\rho_{xx}$  data thus obtained were fitted to the exponential expression leading to a value  $\kappa' = 0.51 \pm 0.05$  (see Fig. 3). Transformations like this generally lead to less quality data. Nevertheless, the results in Fig. 3 indicate that different exponents can be extracted from the same experimental data.

We next address the origin of this difference. First we point out that the transport data of the  $PI$  transition are accurately described by writing

$$\rho_{xx}(\nu, T) = \rho^*(T) \exp(-T^{-\kappa'} \Delta\nu). \quad (2)$$

Here  $\rho^*$  denotes the *critical* resistance. It can be written as  $\rho^* = \sigma_{xx}^*/[(\sigma_{xx}^*)^2 + 1/4]$  where  $\sigma_{xx}^*$  stands for the peak in  $\sigma_{xx}$ . Both quantities are weakly dependent on  $T$  and, quite surprisingly, this  $T$  dependence is not simply *irrelevant* as thought previously. It is, in fact, *marginal*, and we next show that it accounts for the difference in the observed exponents.

Following Eq. (1) we can relate  $\rho_{xx}(\Delta\nu)$  to  $\rho_{xx}(-\Delta\nu)$  such that we can write the ratio as  $\rho_{xx}(\Delta\nu)/\rho_{xx}(-\Delta\nu) = \exp(-2 T^{-\kappa'} \Delta\nu)$ . As a good check upon the validity of this result we have fitted the exponential on the right-hand side to the experimental data which were inserted into the left-hand side. The same numerical value  $\kappa' = 0.55 \pm 0.05$  was obtained indicating once more that Eq. (1) represents the fundamental symmetry in the problem. From the ratio we obtain the following renormalization group

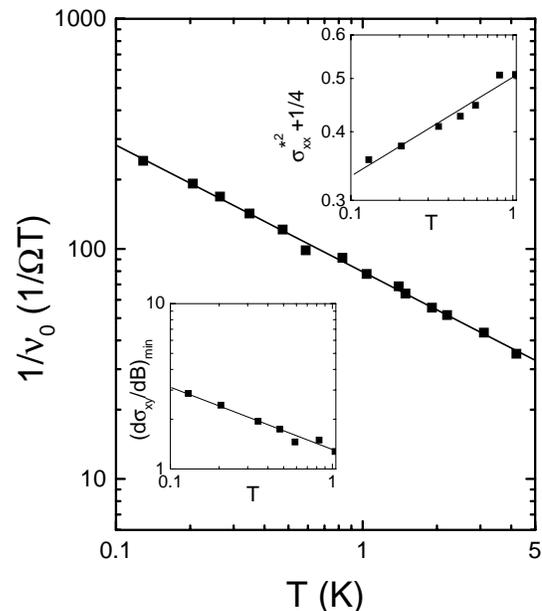


FIG. 4.  $1/\nu_0$  vs  $T$  for the  $PI$  transition (full squares,  $\kappa' = 0.55$ ). Upper inset:  $\sigma_{xx}^{*2} + 1/4$  vs  $T$ . The slope of the straight line equals 0.15. Lower inset:  $(\frac{d\sigma_{xy}}{dB})_{\min}$  vs  $T$ . The slope of the straight line equals 0.43.

equation for small  $\theta = \sigma_{xy} - 1/2$ :

$$\frac{d\theta}{d\ln T} = -\kappa\theta; \quad \kappa = \kappa' - \frac{d\ln(\sigma_{xx}^{*2} + 1/4)}{d\ln T}. \quad (3)$$

Equation (3) shows how a relatively weak  $T$  dependence in  $\sigma_{xx}^*$  can lead to different exponents extracted from different quantities. In Fig. 4 we replot  $1/\nu_0$  vs  $T$  on a log-log scale. The solid line gives  $\kappa' = 0.55$ . In the upper inset we plot the low  $T$  data for  $\ln[(\sigma_{xx}^*)^2 + 1/4]$  vs  $\ln T$  and obtain a slope of  $0.15 \pm 0.03$ . According to Eq. (3) we have  $\kappa \approx 0.40$ . This value should be compared with  $\kappa = 0.43$  extracted from the low  $T$  dependence of  $(d\sigma_{xy}/dB)_{\min}$  (Fig. 4).

A naive interpretation of the *marginal* dependence of  $\sigma_{xx}^*$  on  $T$  is that the electron gas has not yet fully developed criticality. This would mean that a much lower  $T$  is necessary before the critical fixed point is truly reached. However, it is important to stress that the small changes in  $\sigma_{xx}^*$  observed at low  $T$  are most likely the result of macroscopic inhomogeneities in the sample. One way of showing this is by writing Eq. (2) as

$$\rho_{xx}(\Delta\nu, T) = e^{-[\Delta\nu - \delta\nu_c(T)]/\nu_0(T)}. \quad (4)$$

Here the shift in the critical filling fraction ( $\delta\nu_c$ ) and the critical resistance ( $\rho^*$ ) are related through  $\rho^*(T) = e^{\delta\nu_c/\nu_0}$ . This shift is next to be compared to the difference ( $\delta\nu_c$ ) as it is obtained from the definitions  $\sigma_{xy} = 1/2$  and  $d\sigma_{xx}/dB = 0$ . In the lower inset of Fig. 2 we plot the  $\delta\nu_c/\nu_c$  or, equivalently,  $\delta B_c/B_c$  with varying  $T$  for both cases. The two effects are comparable.

Notice that the uncertainty  $\delta\nu_c/\nu_c$  in the definition of  $\nu_c$  clearly shows the effect of macroscopic inhomogeneities (in electron density) which cause  $\nu_c$  to be slightly different in the different regions of the sample where the  $\rho_{xx}$  and  $\rho_{xy}$  are being probed. Figure 2 therefore indicates that the weak or *marginal*  $T$  dependence of  $\sigma_{xx}^*$  is, in fact, an *inhomogeneity* effect. This lack of universality in  $\sigma_{xx}^*$  also shows up in the different data sets taken at different experimental runs. After heating the system up to room  $T$  and then cooling down again one usually finds that the  $B_c$  has shifted (indicating that the electron density has changed) along with a shift in  $\sigma_{xx}^*$  (indicating that the inhomogeneity profile of the density has changed).

In summary, we can say that the *PP* and *PI* transitions are the same. This is in complete agreement with the predictions of the renormalization theory [2]. We have shown that the critical conductance  $\sigma_{xx}^*$  as well as the exponent of the *PI* transition are weakly affected by the (weak) macroscopic inhomogeneities in the sample. Our data retain fundamental aspects such as the electron-hole symmetry in the  $\sigma_{xx}, \sigma_{xy}$  diagram. It is important that this symmetry is not confused with the statement of duality [17] which is, in fact, not verified by our experiments.

The marginal  $T$  dependence in  $\sigma_{xx}^*$  is common to both the *PP* and *PI* transitions in our samples. This was previously also observed [1]. It is important to note that Eq. (3), upon modification, is applicable to the *PP* tran-

sitions as well. For example, for the  $2 \rightarrow 1$  transition Eq. (3) is modified according to  $\kappa' - \kappa = d\ln(\sigma_{xx}^{*2} + 9/4)/d\ln T$ . By inserting the  $\sigma_{xx}^*$  data we find in this case  $\kappa' - \kappa < 0.01$  which is well within the experimental error. This result explains why a single exponent  $\kappa = \kappa' = 0.42 \pm 0.04$  was previously extracted from the  $2 \rightarrow 1$  transition as well as from the higher Landau level experimental data over a wide range in  $T$  (4 K–20 mK). By combining the results for the *PP* and *PI* transitions we conclude that  $\kappa = 0.42$  stands for the *universal* critical exponent of the quantum phase transition. The numerical value  $\kappa' = 0.55$ , on the other hand, is the result of macroscopic inhomogeneities. Following Eq. (3) it represents an *effective* exponent. A deeper understanding of the inhomogeneity effects demands a commitment to research specifically grown InGaAs/InP heterostructures.

This research was supported in part by the Dutch Science Foundation FOM, by the EC under Grant No. FMGECT950079, by the National Science Foundation under Grants No. PHY94-07194 and No. DMR-9311091, and INTAS Grant No. 96-0580.

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